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Geometry of type II common sector $N = 2$ backgrounds

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Abstract: We describe the geometry of all type II common sector backgrounds with two supersymmetries. In particular, we determine the spacetime geometry of those supersymmetric backgrounds for which each copy of the Killing spinor equations admits a Killing spinor. The stability subgroups of both Killing spinors are $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$, $\mathrm{SU}(4) \ltimes \mathbb{R}^8$ and G_2 for IIB backgrounds, and Spin(7), SU(4) and $G_2 \ltimes \mathbb{R}^8$ for IIA backgrounds. We show that the spacetime of backgrounds with spinors that have stability subgroup $K \ltimes \mathbb{R}^8$ is a pp-wave propagating in an eight-dimensional manifold with a K-structure. The spacetime of backgrounds with K-invariant Killing spinors is a fibre bundle with fibre spanned by the orbits of two commuting null Killing vector fields and base space an eight-dimensional manifold which admits a K-structure. Type II T-duality interchanges the backgrounds with K- and $K \ltimes \mathbb{R}^8$-invariant Killing spinors. We show that the geometries of the base space of the fibre bundle and the corresponding space in which the pp-wave propagates are the same. The conformal symmetry of the world-sheet action of type II strings propagating in these $N = 2$ backgrounds can always be fixed in the light-cone gauge.

Keywords: Supergravity Models, Superstring Vacua, Superstrings and Heterotic Strings, Flux compactifications.

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1. Introduction

The type II common sector is a consistent truncation of type II supergravities. The bosonic fields are the metric g, dilaton Φ and three-form field strength H, $dH = 0$. The Killing spinor equations in the string frame are

$$\begin{aligned} \hat{\nabla}\hat{\epsilon} &= 0\,, \qquad \left(d\Phi - \frac{1}{2}H\right)\hat{\epsilon} = 0\,, \\ \check{\nabla}\check{\epsilon} &= 0\,, \qquad \left(d\Phi + \frac{1}{2}H\right)\check{\epsilon} = 0\,, \end{aligned} \tag{1.1}$$

where the Killing spinors $\hat{\epsilon}$ and $\check{\epsilon}$ are Majorana-Weyl of the same (IIB) or opposite chirality (IIA), and $\hat{\nabla} = \nabla + \frac{1}{2}H$ and $\check{\nabla} = \nabla - \frac{1}{2}H$ are metric connections with torsion[1] given by the NS⊗NS three-form H. The gravitino and dilatino Killing spinor equations are two copies of those of the heterotic string differing by the sign of H. Let $(\hat{N}, \check{N})$ be the number of Killing spinors and $(\hat{G}, \check{G})$ be their stability subgroups in $\mathrm{Spin}(9,1)$ for each copy, respectively. The total number of Killing spinors of a background is $N = \hat{N} + \check{N}$. If $\check{N} = 0$, then the Killing spinor equations reduce to those of the heterotic string. Consequently, the associated supersymmetric backgrounds are those of the heterotic string and a systematic investigation of their geometries can be found in [1] using the spinorial geometry approach of [2]. There is extensive previous work on the geometry of heterotic and type II common sector supersymmetric backgrounds, see e.g. [3]–[19]. Similarly, if $\hat{N} = 0$, the geometries of the common sector backgrounds can also be recovered from those of the heterotic string. However, if both sectors admit a non-trivial Killing spinor, $\hat{N}, \check{N} \neq 0$, new geometries arise. The conditions that each copy of Killing spinor equations imposes on the geometry are those of, or can be constructed from, the associated heterotic string backgrounds with $\hat{N}$ and $\check{N}$ Killing spinors and with stability subgroups $\hat{G}$ and $\check{G}$, respectively. Furthermore, the geometry of the spacetime depends on the conditions that arise from both copies of the Killing spinor equations and in particular on the stability subgroup, $G = \hat{G} \cap \check{G}$, of all Killing spinors.[2] In turn, G depends on the way that $\hat{G}$ and $\check{G}$ are embedded in $\mathrm{Spin}(9,1)$ up to a $\mathrm{Spin}(9,1)$-conjugation. There are many ways to embed $\hat{G}$ and $\check{G}$ in $\mathrm{Spin}(9,1)$ which lead to inequivalent geometries for the spacetime. In [1] it was argued that all these new geometries appear because, unlike the case of the heterotic string, the gauge

[1] In the literature, these connections are denoted with ∇^+ and ∇^-, respectively. We have introduced a different notation because later we use $-$ and $+$ labels to denote light-cone directions.

[2] The use of the stability subgroups of the spinors in the context of supersymmetric solutions has been suggested in [20].

group $\mathrm{Spin}(9,1)$ of the common sector Killing spinor equations is a proper subgroup of the holonomy group $\mathrm{Spin}(9,1) \times \mathrm{Spin}(9,1)$ of the supercovariant connection of the gravitino Killing spinor equations.

All $N = 1$ supersymmetric common sector backgrounds[3] are embeddings of $N = 1$ supersymmetric heterotic backgrounds and therefore their geometry has already been described in [1]. As we have already mentioned, the geometry of $N = 2$ backgrounds with either $\hat{N} = 0$ or $\check{N} = 0$ are also embeddings of heterotic string backgrounds with two Killing spinors. In this paper, we shall examine the geometry of type II common sector $N = 2$ backgrounds with $\hat{N} = \check{N} = 1$. It has been shown in [1] that there are three classes of such type IIB backgrounds characterized by the spinor stability subgroups $G = \mathrm{Spin}(7) \ltimes \mathbb{R}^8$, $\mathrm{SU}(4) \ltimes \mathbb{R}^8$ and G_2. We shall show that there are also three classes of IIA backgrounds characterized by the spinor stability subgroups $G = \mathrm{Spin}(7)$, $\mathrm{SU}(4)$ and $G_2 \ltimes \mathbb{R}^8$. It is clear that there are two types of stability subgroups K and $K \ltimes \mathbb{R}^8$, where K is a compact group, and the type II N=2 backgrounds come in pairs. If a IIA background has stability subgroup $G = K$, then the corresponding IIB background has stability subgroup $G = K \ltimes \mathbb{R}^8$ and vice-versa. All the backgrounds with stability subgroup $G = K \ltimes \mathbb{R}^8$ admit a null ∇-parallel vector field and therefore the spacetime is a pp-wave propagating in an eight-dimensional space B equipped with a K-structure. Equivalently, the spacetime can be thought of as a Lorentzian deformation family of B. In particular, the metric and three-form are

$$\begin{aligned} ds^2 &= 2dv(du + Vdv + n) + \delta_{ij}e^i e^j \,, \\ H &= \frac{1}{2}H^{\mathfrak{k}}_{-ij}e^- \wedge e^i \wedge e^j + \frac{1}{2}H^{\mathfrak{k}^\perp}_{-ij}e^- \wedge e^i \wedge e^j + \frac{1}{3!}H_{ijk}e^i \wedge e^j \wedge e^k \,, \\ \Phi &= \Phi(v, y) \,, \end{aligned} \tag{1.2}$$

where the parallel vector field is $X = \partial/\partial u$ and the deformed manifold B is defined by $u, v = \text{const}$. Let $\mathfrak{k} \subset \Lambda^2(\mathbb{R}^8)$ be the Lie algebra of K and $\Lambda^2(\mathbb{R}^8) = \mathfrak{k} \oplus \mathfrak{k}^\perp$. The Killing spinor equations specify all the components of H in terms of the geometry apart from $H^{\mathfrak{k}}_{-ij}$ which remains undetermined. The geometry of the eight-dimensional deformed manifold is constrained. In particular for $K = \mathrm{Spin}(7)$, B is a $\mathrm{Spin}(7)$ manifold, $\mathrm{hol}(\tilde{\nabla}) \subseteq \mathrm{Spin}(7)$, and $H_{ijk} = 0$. For $K = \mathrm{SU}(4)$, B is an almost hermitian manifold with an $\mathrm{SU}(4)$ structure, the canonical bundle of B admits a trivialization and the associated (4,0)-form is parallel with respect to the Levi-Civita connection. The classes W_1 and W_4 are related to the trivialization of the canonical bundle, and W_2 is related to the W_3 class. For $K = G_2$, B admits a vector field Z which is rotation free but not Killing. There are 2^{10} G_2-structures on an eight-dimensional manifold and we specify the one that is associated with $N = 2$ supersymmetry. The components H_{ijk} of the torsion are determined in terms of Z and its derivatives. In addition, we analyze the integrability conditions of the Killing spinor equation and we show that all the field equations are satisfied provided that one imposes the Bianchi identity of H, $dH = 0$, and the $E_{--} = 0$ component of the Einstein equations.

[3]It suffices to know the geometry of $\hat{N} = 1$, $\check{N} = 0$ backgrounds. This is because the geometry of $\hat{N} = 0$, $\check{N} = 1$, backgrounds can be easily be determined from that of $\hat{N} = 1$, $\check{N} = 0$ backgrounds, e.g. in IIB common sector one has to set $H \to -H$.

The backgrounds with stability subgroups K are (locally) fibre bundles of rank two over an eight-dimensional manifold B. The fibre directions are spanned by the orbits of two commuting null $\hat{\nabla}$- and $\check{\nabla}$-parallel vector fields X, Y. The metric and torsion can be written as

$$\begin{aligned} ds^2 &= 2f^4(dv+m)(du+n) + \delta_{ij}e^ie^j \,, \\ H &= d(e^- \wedge e^+) + \frac{1}{3!}H_{ijk}e^i \wedge e^j \wedge e^k \,, \\ \Phi &= \Phi(y) \,, \end{aligned} \tag{1.3}$$

where $X = \partial/\partial u$ and $Y = \partial/\partial v$. All the components of the torsion H are determined in terms of the geometry of spacetime. In particular, if $G = \mathrm{Spin}(7)$, $H_{ijk} = 0$. If the stability subgroup of the spinors in K, we find that the geometry of the base space B is exactly the same as that of the deformed manifold B for the backgrounds with $K \ltimes \mathbb{R}^8$-invariant spinors described above. As in the previous case, we analyze the integrability conditions of the Killing spinor equations and show that all the field equations are satisfied provided that one imposes the Bianchi identity of H, $dH = 0$ and $LH_{-+} = 0$.

We also investigate the dynamics of fundamental string probes in the above supersymmetric backgrounds. As an application of the geometry of $\hat{N} = \check{N} = 1$ backgrounds, we show that conformal symmetry of the world-sheet action of strings propagating in $\hat{N}, \check{N} \geq 1$ supersymmetric backgrounds can always be fixed in the light-cone gauge. In addition, we give the bosonic part of the light-cone world-sheet actions. Then we investigate the relation between spacetime supersymmetry and the world-sheet chiral W-type of currents of [24] for string probes.

This paper has been organized as follows: in section 2, we give the representatives of the Killing spinors and their stability subgroups in Spin(9, 1), and describe some of the properties of the Killing spinor form bilinears. In section 3, we solve the Killing spinor equations of $\hat{N} = \check{N} = 1$ type IIB supersymmetric backgrounds and we investigate the associated geometries. In section 4, we solve the Killing spinor equations of $\hat{N} = \check{N} = 1$ type IIA supersymmetric backgrounds and we investigate the associated geometries. In section 5, we give the light-cone action of strings in type II backgrounds with $\hat{N}, \check{N} \geq 1$ supersymmetry and in section 6, we present our conclusions. In appendix A, we summarize the type II Killing spinor equations, their integrability conditions and the field equations of the theory. In appendix B, we evaluate the type IIA Killing spinor equations on a basis in the space of negative chirality spinors. This together with the results of [1] provides all the data one needs for the systematics of type II common sector. In appendix C, we summarize some of the properties of eight-dimensional manifolds with SU(4) and G_2 geometries.

2. Spinors, holonomy and forms

2.1 Holonomy, gauge symmetry and parallel spinors

The spinor bundle of the IIB common sector is $S^+ \oplus S^+$ and the Killing spinor equations are two copies of those of the heterotic string differing by the sign of the NS⊗ NS three-form field strength, see (1.1). In particular, the gravitino Killing spinor equation is a parallel transport equation for the $\mathrm{Spin}(9,1)\times\mathrm{Spin}(9,1)$ connection $\hat{\nabla}\oplus\check{\nabla}$. The gauge group that preserves the Killing spinor equations is $\mathrm{Spin}(9,1)$. This is in contrast to the heterotic case where the holonomy group of the connection of the gravitino Killing spinor equation coincides with the gauge group. Since the gauge group of the common sector is the same as that of the heterotic string but the dimension of the space of spinors is twice as large, there are many more cases to investigate than those examined for the heterotic string in [1].

The systematics of type IIB common sector Killing spinor equations can be read off from those of the heterotic string in [1]. This is because the representation of the Killing spinors is two copies of that of the heterotic string, i.e. $\Delta^+_{\mathbf{16}}\oplus\Delta^+_{\mathbf{16}}$. In particular, the linear system of the first copy of the Killing spinor equations is exactly the same as that of the heterotic string. The linear system of the second copy can be read off from that of the heterotic string by setting $H\to -H$.

As a consequence, the conditions that each copy of the Killing spinor equations imposes on the geometry of spacetime can be found from those of the heterotic string in [1]. In particular, $\mathrm{hol}(\hat{\nabla})\subseteq\hat{G}$ and $\mathrm{hol}(\check{\nabla})\subseteq\check{G}$, where $\hat{G}$ and $\check{G}$ are the stability subgroups of the Killing spinors of each copy in $\mathrm{Spin}(9,1)$. However, the geometry of spacetime depends on the conditions of both copies and in particular of the stability subgroup G of all Killing spinors in $\mathrm{Spin}(9,1)$. In turn, G depends on the embedding of $\hat{G}$ and $\check{G}$ in $\mathrm{Spin}(9,1)$ up to a conjugation with a $\mathrm{Spin}(9,1)$ gauge transformation.

To illustrate the above analysis, consider the $N=2$ IIB common sector backgrounds with $\hat{N}=\check{N}=1$. It has been shown in [1] that the Killing spinors can be chosen as

$$\hat{\epsilon}=f(1+e_{1234})\,,\qquad \check{\epsilon}=g_1(1+e_{1234})+ig_2(1-e_{1234})+g_3(e_{15}+e_{2345})\,,\tag{2.1}$$

where f, g_1, g_2 and g_3 are spacetime functions. Both the spinor $\hat{\epsilon}$ and $\check{\epsilon}$ are representatives of the $\mathrm{Spin}(7)\ltimes\mathbb{R}^8$ orbit of $\mathrm{Spin}(9,1)$ in the Majorana-Weyl representation $\Delta^+_{\mathbf{16}}$. Therefore $\hat{G}=\check{G}=\mathrm{Spin}(7)\ltimes\mathbb{R}^8$. The second spinor $\check{\epsilon}$ has been constructed by decomposing $\Delta^+_{\mathbf{16}}$ under $\mathrm{Spin}(7)\ltimes\mathbb{R}^8$ representations and taking suitable representatives of the orbits. The stability subgroup of both spinors depends on the coefficients g_1, g_2 and g_3. Note that if $g_3\neq 0$, one can set $g_1=g_2=0$ by acting with an $\mathbb{R}^8\subset\mathrm{Spin}(7)\ltimes\mathbb{R}^8$ transformation which stabilizes $\hat{\epsilon}$, see [1] for details. In table 1, we summarize the stability subgroups of the Killing spinors of $N=2$ IIB common sector backgrounds.

The Killing spinor equations of the type IIA common sector are somewhat different from those of the type IIB common sector. This is because the type IIA spinor bundle is $S^+\oplus S^-$ and therefore is not just two copies of that of the heterotic string. Nevertheless, the systematics of the IIA common sector Killing spinor equations can again be read off from those of the heterotic string in [1]. In particular, the linear system for the first copy is identical to that of the heterotic string. The linear system of the second copy of the Killing spinor equations has also some similarities with that of the heterotic string and is given in appendix B.

The analysis of the stability subgroups of spinors in the IIA common sector and their relation to the geometry is similar to the one we have presented for the IIB common sector. However there are some differences due to the different representations of the Killing spinors. This can be seen in the $N=2$ backgrounds with $\hat{N}=\check{N}=1$. One can

IIB, $N=2$	$\hat{\mathrm{G}}$	$\check{\mathrm{G}}$	G	Spinors
	$\mathrm{SU}(4) \ltimes \mathbb{R}^8$	-	$\mathrm{SU}(4) \ltimes \mathbb{R}^8$	$-$
	G_2	-	G_2	$-$
	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$g_1 \neq 0, g_2 = g_3 = 0$
	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$\mathrm{SU}(4) \ltimes \mathbb{R}^8$	$g_1, g_2 \neq 0, g_3 = 0$
	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	$\mathrm{Spin}(7) \ltimes \mathbb{R}^8$	G_2	$g_1 = g_2 = 0,\ g_3 \neq 0$

Table 1: There are five classes of IIB common sector backgrounds with two supersymmetries up to $\mathrm{Spin}(9,1)$ gauge transformations. These are denoted with the stability subgroups $\hat{G}$, $\check{G}$ and G of the Killing spinors. In all cases $\mathrm{hol}(\hat{\nabla}) \subseteq \hat{G}$ and $\mathrm{hol}(\check{\nabla}) \subseteq \check{G}$. The entries $-$ denote the cases for which the sector associated with the $\check{\nabla}$ connection does not admit Killing spinors. The last column gives the restrictions on the parameters of the second Killing spinor in (2.1).

again choose $\hat{\epsilon} = f(1 + e_{1234})$. It remains to find representatives of the second spinor in $\Delta^-_{\mathbf{16}}$ up to $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$ transformations which is the stability subgroup of $\hat{\epsilon}$ in $\mathrm{Spin}(9,1)$. One can show that under $\mathrm{Spin}(7)$, $\Delta^-_{\mathbf{16}}$ decomposes as $\mathbb{R} \oplus \mathbb{R}^7 \oplus \Delta^+_{\mathbf{8}}$. The spinor that represents the singlet in the decomposition is proportional to $e_5 + e_{12345}$. The stability subgroup of $1 + e_{1234}$ and $e_5 + e_{12345}$ is $\mathrm{Spin}(7)$ rather than $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$ that appears in the IIB case. The second spinor can be written as $\check{\epsilon} = g_1(e_5 + e_{12345}) + \epsilon_2 + \epsilon_3$, where ϵ_2 and ϵ_3 lie in the seven and eight-dimensional representations respectively. If the component ϵ_3 vanishes, then $\mathrm{Spin}(7)$ acts transitively on the sphere in $\mathbb{R}^7$ and so the representative can be chosen to lie in any direction. In particular, one can choose as the second spinor $\check{\epsilon} = g_1(e_5 + e_{12345}) + ig_2(e_5 - e_{12345})$. The stability subgroup of both spinors is $\mathrm{SU}(4)$. Next suppose that ϵ_3 does not vanish. To choose ϵ_3 observe that $\mathrm{Spin}(7)$ acts transitively on the sphere in $\Delta^+_{\mathbf{8}}$ with stability subgroup G_2. In turn G_2 acts transitively on the sphere in $\mathbb{R}^7$ with stability subgroup $\mathrm{SU}(3)$. To summarize, we have found that one can always choose the two Killing spinors as

$$\hat{\epsilon} = f(1 + e_{1234})\,, \qquad \check{\epsilon} = g_1(e_5 + e_{12345}) + ig_2(e_5 - e_{12345}) + g_3(e_1 + e_{234})\,. \tag{2.2}$$

There is one type of orbit of $\mathrm{Spin}(9,1)$ in $\Delta^-_{\mathbf{16}}$ with stability subgroup $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$, thus the stability subgroup of either $\hat{\epsilon}$ or $\check{\epsilon}$ is $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$, i.e. $\hat{G} = \mathrm{Spin}(7) \ltimes \mathbb{R}^8$ and $\check{G} = \mathrm{Spin}(7) \ltimes \mathbb{R}^8$. To find the stability subgroups of both $\hat{\epsilon}$ and $\check{\epsilon}$, observe that if $g_1 \neq 0$, the element

$$g = e^{\frac{1}{\sqrt{2}(g_1^2+g_2^2)}(g_1 g_3 \Gamma^1 \Gamma^- - g_2 g_3 \Gamma^6 \Gamma^-)} \tag{2.3}$$

of $\mathrm{Spin}(9,1)$ leaves invariant $\hat{\epsilon}$ and transforms the spinor $g_1(e_5 + e_{12345}) + ig_2(e_5 - e_{12345})$ to $\check{\epsilon}$ in (2.2), and similarly for $g_2 \neq 0$. Therefore, if either $g_1 \neq 0$ or $g_2 \neq 0$, one can always choose $g_3 = 0$. This is in analogy with a similar result in type IIB [1]. The stability subgroups of $\hat{\epsilon}$ and $\check{\epsilon}$ in the IIA common sector are summarized in table 2.

The stability subgroups of the spinors in the type IIA and type IIB common sector backgrounds with two supersymmetries are different in all $\hat{N}, \check{N} \neq 0$ cases. However, they are related by the interchange $K \leftrightarrow K \ltimes \mathbb{R}^8$, where $K = \mathrm{Spin}(7), \mathrm{SU}(4)$ and G_2. We shall argue later that this is due to the type II T-duality.

IIA, $N=2$	$\hat{\mathrm{G}}$	$\check{\mathrm{G}}$	G	Spinors
	$SU(4)\ltimes\mathbb{R}^8$	-	$SU(4)\ltimes\mathbb{R}^8$	-
	G_2	-	G_2	-
	$Spin(7)\ltimes\mathbb{R}^8$	$Spin(7)\ltimes\mathbb{R}^8$	Spin(7)	$g_1\neq 0, g_2=g_3=0$
	$Spin(7)\ltimes\mathbb{R}^8$	$Spin(7)\ltimes\mathbb{R}^8$	SU(4)	$g_1, g_2\neq 0, g_3=0$
	$Spin(7)\ltimes\mathbb{R}^8$	$Spin(7)\ltimes\mathbb{R}^8$	$G_2\ltimes\mathbb{R}^8$	$g_1=g_2=0, g_3\neq 0$

Table 2: There are five classes of IIA common sector backgrounds with two supersymmetries. These are denoted with the stability subgroups $\hat{G}$, $\check{G}$ and G of the Killing spinors. In all cases $\mathrm{hol}(\hat{\nabla})\subseteq\hat{G}$ and $\mathrm{hol}(\check{\nabla})\subseteq\check{G}$. The entries $-$ denote the cases for which the copy associated with the $\check{\nabla}$ connection does not admit Killing spinors. The last column gives the restrictions on the parameters of the second Killing spinor in (2.2).

2.2 Spacetime form bilinears

There are three kinds of spacetime form spinor bilinears that one can construct for the type II common sector. One kind is the form bilinears that are constructed from $\hat{\nabla}$-parallel spinors. Another kind are those that are constructed from $\check{\nabla}$-parallel spinors and the third kind are those which are constructed from one $\hat{\nabla}$- and one $\check{\nabla}$-parallel spinor. We denote these bilinears with $\hat{\alpha}$, $\check{\alpha}$ and α, respectively. It is clear that

$$\hat{\nabla}_A\hat{\alpha}=0\,,\qquad \check{\nabla}_A\check{\alpha}=0\,. \tag{2.4}$$

The bilinears α are not apparently parallel with neither $\hat{\nabla}$ nor $\check{\nabla}$ connections. Instead, one finds that

$$\begin{aligned}\nabla_A\alpha &= \frac{1}{k!}B(\nabla_A\hat{\psi},\Gamma_{B_1\dots B_k}\check{\eta})e^{B_1}\wedge\cdots\wedge e^{B_k}+\frac{1}{k!}B(\hat{\psi},\Gamma_{B_1\dots B_k}\nabla_A\check{\eta})e^{B_1}\wedge\cdots\wedge e^{B_k}\\ &= -\frac{1}{8\cdot k!}H_{AC_1C_2}B(\hat{\psi},\{\Gamma^{C_1C_2},\Gamma_{B_1\dots B_k}\}\check{\eta})e^{B_1}\wedge\ldots\wedge e^{B_k}\,.\end{aligned} \tag{2.5}$$

The computation of the form bilinears can be done as for the heterotic string [1].

A special class of spinor bilinears are the one-forms. If $\hat{\kappa}_X$ and $\check{\kappa}_Y$ are parallel one-forms with respect to the connections $\hat{\nabla}$ and $\check{\nabla}$, respectively, then the associated vector fields $\hat{X}$ and $\check{Y}$ are Killing

$$\mathcal{L}_{\hat{X}}g=0\,,\qquad \mathcal{L}_{\check{Y}}g=0\,, \tag{2.6}$$

and

$$d\hat{\kappa}_X=i_{\hat{X}}H\,,\qquad d\check{\kappa}_Y=-i_{\check{Y}}H\,. \tag{2.7}$$

Since $dH=0$, clearly $\mathcal{L}_{\hat{X}}H=\mathcal{L}_{\check{Y}}H=0$.

As in the case of the heterotic string, the commutators of $\hat{\nabla}$- and $\check{\nabla}$-parallel vector fields are determined in terms of H. In particular, one finds that

$$\begin{aligned}&[\hat{X},\hat{Y}]=i_{\hat{X}}i_{\hat{Y}}H^Ae_A\,,\qquad [\check{X},\check{Y}]=-i_{\check{X}}i_{\check{Y}}H^Ae_A\,,\\ &[\hat{X},\check{Y}]=0\,.\end{aligned} \tag{2.8}$$

Note that the commutator of $\hat{\nabla}$-parallel with $\check{\nabla}$-parallel vector fields vanishes. This property is widely applicable because in all $N\geq 2$, $\hat{N},\check{N}\geq 1$, backgrounds there is at least

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one $\hat{\nabla}$-parallel and one $\check{\nabla}$-parallel vector field. However, these vector fields are not always linearly independent. In addition if $\hat{X}, \hat{Y}$ and $\check{X}, \check{Y}$ are $\hat{\nabla}$- and $\check{\nabla}$-parallel, then $[\hat{X}, \hat{Y}]$ and $[\check{X}, \check{Y}]$ are also $\hat{\nabla}$- and $\check{\nabla}$-parallel, respectively [1].

Suppose that $\hat{\alpha}$ and $\check{\alpha}$ are k-forms, and that $\hat{X}$ and $\check{X}$ are vector fields. Then, one can show using $\hat{\nabla}\hat{\alpha} = \hat{\nabla}\hat{X} = \check{\nabla}\check{\alpha} = \check{\nabla}\check{X} = 0$ that

$$\begin{aligned} (\mathcal{L}_{\hat{X}}\hat{\alpha})_{A_1\dots A_k} &= k(-1)^k (i_{\hat{X}}H)^B{}_{[A_1}\hat{\alpha}_{A_2\dots A_k]B}\,, \\ (\mathcal{L}_{\check{X}}\check{\alpha})_{A_1\dots A_k} &= -k(-1)^k (i_{\check{X}}H)^B{}_{[A_1}\check{\alpha}_{A_2\dots A_k]B}\,, \\ \mathcal{L}_{\hat{X}}\check{\alpha} &= \mathcal{L}_{\check{X}}\hat{\alpha} = 0\,. \end{aligned} \tag{2.9}$$

Therefore $\mathcal{L}_{\hat{X}}\hat{\alpha} = \mathcal{L}_{\check{X}}\check{\alpha} = 0$, iff the rotations of $\hat{X}$ and $\check{X}$, $i_{\hat{X}}H$ and $-i_{\check{X}}H$, leave invariant the forms $\hat{\alpha}$ and $\check{\alpha}$, respectively.

3. IIB $N = 2$ backgrounds

3.1 Backgrounds with $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$-invariant Killing spinors

3.1.1 Supersymmetry conditions

The conditions that the Killing spinor equations impose on the geometry of spacetime can be easily read off from those of the heterotic string for backgrounds with one supersymmetry. As we have explained, the $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$-invariant Killing spinors can be chosen as

$$\hat{\epsilon} = f(1 + e_{1234})\,, \qquad \check{\epsilon} = g(1 + e_{1234})\,. \tag{3.1}$$

The conditions that are implied by the spinor $\hat{\epsilon}$ are exactly those of the $N = 1$ heterotic string backgrounds found in [1]. The conditions that are implied by the spinor $\check{\epsilon}$ can be read off from those of the $N = 1$ heterotic string backgrounds after substituting $H \to -H$. Because of this, we shall not elaborate on the derivation of the linear system associated with the Killing spinor equations. The relation between linear systems and Killing spinor equations is explained in [21]. The linear system can be solved to give

$$\begin{aligned} & g = f\,, \qquad 2\partial_A \log f + \Omega_{A,-+} = 0\,, \qquad \Omega_{A,\alpha}{}^{\alpha} = 0 \\ & \Omega_{A,\bar{\alpha}\bar{\beta}} - \frac{1}{2}\Omega_{A,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar{\alpha}\bar{\beta}} = 0\,, \qquad \Omega_{A,+\alpha} = 0\,, \\ & \partial_\alpha\Phi = \partial_+\Phi = 0\,, \qquad H_{ijk} = 0\,, \qquad H_{+AB} = 0\,, \\ & H_{-\alpha}{}^{\alpha} = 0\,, \qquad H_{-\bar{\alpha}\bar{\beta}} - \frac{1}{2}H_{-\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar{\alpha}\bar{\beta}} = 0\,, \end{aligned} \tag{3.2}$$

where we have used that f, g are defined up to an overall constant scale. The last three conditions above imply that the only non-vanishing components of the flux H are H_{-ij} and they take values in $\mathfrak{spin}(7) \subset \mathfrak{so}(8) = \Lambda^2(\mathbb{R}^8)$. Using a spin gauge transformation in the direction Γ_{-+}, one can choose the gauge $f = 1$, which implies $g = 1$ as can be seen from the above equations. In this gauge, the Levi-Civita connection satisfies

$$\Omega_{A,+B} = 0\,. \tag{3.3}$$

This simplifies the investigation of the geometry. The geometry of spacetime is independent of the choice of gauge.

3.1.2 The geometry of spacetime

The linearly independent spacetime form bilinears constructed from the Killing spinors $\hat{\epsilon} = f(1 + e_{1234}) = \check{\epsilon}$, after an appropriate normalization, are

$$\hat{\kappa} = \kappa(\hat{\epsilon}, \hat{\epsilon}) = f^2(e^0 - e^5)\,, \qquad \hat{\tau} = \tau(\hat{\epsilon}, \hat{\epsilon}) = f^2(e^0 - e^5) \wedge \phi\,, \tag{3.4}$$

where

$$\phi = \mathrm{Re}\chi - \frac{1}{2}\omega \wedge \omega\,, \tag{3.5}$$

is a Spin(7)-invariant form, $\chi = (e^1 + ie^6) \wedge (e^2 + ie^7) \wedge (e^3 + ie^8) \wedge (e^4 + ie^9)$ and $\omega = -(e^1 \wedge e^6 + e^2 \wedge e^7 + e^3 \wedge e^8 + e^4 \wedge e^9)$. Clearly $\hat{\alpha} = \check{\alpha}$, where $\hat{\alpha}$ and $\check{\alpha}$ denote collectively all the bilinears constructed from $(\hat{\epsilon}, \hat{\epsilon})$ and $(\check{\epsilon}, \check{\epsilon})$, respectively.

As we have already explained, $\hat{\nabla}\hat{\alpha} = \check{\nabla}\check{\alpha} = 0$. Furthermore, in the gauge $f = g = 1$, $\hat{\alpha} = \check{\alpha} = \alpha$. This implies that α is also parallel with respect to the Levi-Civita connection. In particular, one finds that

$$\nabla_A \kappa_B = 0\,, \qquad \nabla_A \phi_{ijkl} = 0\,, \tag{3.6}$$

where $i, j = 1, 2, 3, 4, 6, 7, 8, 9$. The former condition can also be easily seen from (3.3). Therefore the holonomy of the Levi-Civita connection, ∇, is contained in $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$, $\mathrm{hol}(\nabla) \subseteq \mathrm{Spin}(7) \ltimes \mathbb{R}^8$.

A consequence of this is that the vector field X associated with $\kappa = e^-$ is rotation free. If one adapts coordinates along X, $X = \partial/\partial u$, and uses that X is rotation free, then $e^- = dv$ and the metric, flux and dilaton can be written as

$$\begin{aligned} ds^2 &= 2e^-e^+ + \delta_{ij}e^ie^j = 2dv(du + Vdv + n_I dy^I) + g_{IJ}dy^I dy^J\,, \\ H &= \frac{1}{2}H^{\mathfrak{spin}(7)}_{-ij}\, e^- \wedge e^i \wedge e^j\,, \\ \Phi &= \Phi(v)\,, \end{aligned} \tag{3.7}$$

where all the components of the fields depend on v, y and as is indicated H_{-ij} takes values in $\mathfrak{spin}(7)$.

The spacetime is a pp-wave propagating in an eight-dimensional Spin(7) manifold B given by $u, v = \mathrm{const}$. The holonomy of the Levi-Civita connection of B, $\tilde{\nabla}$, is contained in Spin(7), $\mathrm{hol}(\tilde{\nabla}) \subseteq \mathrm{Spin}(7)$. Alternatively, the spacetime can be thought of as a two-parameter[4] Lorentzian family of eight-dimensional Spin(7) holonomy manifolds B.

3.1.3 Field equations

The integrability conditions of the Killing spinor equations have been given in appendix A. As is well-known, these can be used to find which field equations are implied as a consequence of the Killing spinor equations. A straightforward calculation using the equations in appendix A and the results in appendix B reveals that if one imposes

$$dH = 0\,, \qquad E_{--} = 0\,, \tag{3.8}$$

then all the rest of the field equations are implied. This is the case for all the $\hat{N} = \check{N} = 1$ supersymmetric common sector backgrounds which admit $K \ltimes \mathbb{R}^8$-invariant Killing spinors. As the proof is very similar for the other cases we shall not repeat the analysis in each case.

[4]The geometry of B depends non-trivially only on v.

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3.2 Backgrounds with $\mathrm{SU}(4) \ltimes \mathbb{R}^8$-invariant Killing spinors

3.2.1 Supersymmetry conditions

The $\mathrm{SU}(4) \ltimes \mathbb{R}^8$-invariant Killing spinors can be chosen as

$$\hat{\epsilon} = f(1+e_{1234})\,, \qquad \check{\epsilon} = (g_1+ig_2)1 + (g_1-ig_2)e_{1234}\,. \tag{3.9}$$

The linear system associated with the above Killing spinors can be easily derived from the heterotic supergravity results of [1] so we shall not give further details. The solution of the linear system can be written as

$$\begin{aligned}
&\Omega_{A,+B} = 0\,, \qquad H_{+AB} = 0\,, \qquad \hat{\Omega}_{A,\alpha}{}^{\alpha} = 0,\\
&\partial_A(g_1+ig_2) + \Omega_{A,\alpha}{}^{\alpha}(g_1+ig_2) = 0\,,\\
&(g_1+ig_2)\check{\Omega}_{A,\bar\alpha\bar\beta} - \frac{1}{2}(g_1-ig_2)\check{\Omega}_{A,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&\hat{\Omega}_{A,\bar\alpha\bar\beta} - \frac{1}{2}\hat{\Omega}_{A,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&\partial_+\Phi = 0\,, \qquad \partial_{\bar\alpha}\Phi - \frac{1}{2}H_{\bar\alpha\beta}{}^{\beta} + \frac{1}{6}H_{\beta_1\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = 0\,,\\
&g_1\partial_{\bar\alpha}\Phi + \frac{i}{2}g_2 H_{\bar\alpha\beta}{}^{\beta} = 0\,,
\end{aligned} \tag{3.10}$$

where for simplicity we have chosen the gauge $f=1$ and $\alpha,\beta,\gamma,\delta = 1,2,3,4$. This gauge can always be attained using a local $\mathrm{Spin}(9,1)$ transformation in the direction of Γ_{-+}.

Next observe that the condition $\partial_A(g_1+ig_2)+\Omega_{A,\alpha}{}^{\alpha}(g_1+ig_2)=0$ implies that $\partial_A(g_1^2+g_2^2)=0$. Since the Killing spinors are determined up to an overall constant, we can set $g_1^2+g_2^2=1$. In turn, we can write

$$i\partial_A\lambda + \Omega_{A,\alpha}{}^{\alpha} = 0\,, \qquad g_1+ig_2 = e^{i\lambda}\,. \tag{3.11}$$

Using this and (3.10), we find that some of the components of the fluxes can be expressed in terms of the geometry as

$$\begin{aligned}
&H_{+AB} = 0\,, \qquad H_{\beta_1\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = -6\partial_{\bar\alpha}(\Phi+i\lambda)\,, \qquad H_{A\alpha}{}^{\alpha} = -2i\partial_A\lambda\,,\\
&\partial_{\bar\alpha}[\Phi - \log\cos\lambda] = 0\,, \qquad \partial_+\Phi = 0\,,\\
&H_{A'\bar\alpha\bar\beta} = \frac{2}{e^{2i\lambda}-1}[-(1+e^{2i\lambda})\Omega_{A',\bar\alpha\bar\beta} + \Omega_{A',\beta_1\beta_2}\epsilon^{\beta_1\beta_2}{}_{\bar\alpha\bar\beta}]\,, \qquad A' = -,\gamma\,.
\end{aligned} \tag{3.12}$$

In addition, we find the conditions

$$\begin{aligned}
&\Omega_{A,+B} = 0\,, \qquad i\partial_A\lambda + \Omega_{A,\alpha}{}^{\alpha} = 0\,, \qquad \partial_+\lambda = 0\,, \qquad \Omega_{+,\alpha\beta} = 0\,,\\
&-(1+e^{2i\lambda})[2\Omega_{\bar\beta_1,\bar\beta_2\bar\beta_3} - \Omega_{\bar\beta_3,\bar\beta_1\bar\beta_2} - \Omega_{\bar\beta_2,\bar\beta_3\bar\beta_1}] + 2\Omega_{\bar\beta_1,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\beta_2\bar\beta_3} - \Omega_{\bar\beta_3,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\beta_1\bar\beta_2}\\
&\quad -\Omega_{\bar\beta_2,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\beta_3\bar\beta_2} = 0\,,\\
&\Omega_{\beta,}{}^{\beta}{}_{\bar\alpha} = -\frac{1+2\sin^2\lambda}{\sin 2\lambda}\partial_{\bar\alpha}\lambda\,,\\
&\Omega_{\beta_1,\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = -\frac{e^{i\lambda}}{\sin\lambda}\partial_{\bar\alpha}\lambda\,,
\end{aligned} \tag{3.13}$$

on the geometry of spacetime. If one does not choose the gauge $f=1$, then it is easy to see that $f^{-1}(g_1+ig_2)=e^{i\lambda}$ and that $\Omega_{A,+-}$ does not vanish but is pure gauge. Otherwise the rest of the equations are not affected.

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3.2.2 The geometry of spacetime

The spacetime forms $\hat{\alpha}$ associated with the spinor $\hat{\epsilon}$ have been computed in (3.4). After an appropriate normalization that we suppress, the remaining ones are

$$
\begin{aligned}
&\kappa(\check{\epsilon},\check{\epsilon}) = e^0 - e^5\,, \qquad \kappa(\hat{\epsilon},\check{\epsilon}) = \cos\lambda\,(e^0 - e^5)\,, \qquad \xi(\hat{\epsilon},\check{\epsilon}) = \sin\lambda\,(e^0 - e^5)\wedge\omega\,,\\
&\tau(\check{\epsilon},\check{\epsilon}) = (e^0 - e^5)\wedge\left[\mathrm{Re}(e^{2i\lambda}\chi) - \frac{1}{2}\omega\wedge\omega\right],\\
&\tau(\hat{\epsilon},\check{\epsilon}) = (e^0 - e^5)\wedge\mathrm{Re}\left[e^{i\lambda}(\chi - \frac{1}{2}\omega\wedge\omega)\right],
\end{aligned}
\tag{3.14}
$$

where $\chi = (e^1 + ie^6)\wedge\ldots\wedge(e^4 + ie^9)$ and $\omega = -(e^1\wedge e^6 + e^2\wedge e^7 + e^3\wedge e^8 + e^4\wedge e^9)$. The Hermitian-type of form ω gives rise to an endomorphism I of the tangent bundle of the spacetime which can be identified with an almost complex structure transverse to the light-cone directions. The conditions on the geometry can then be written in a covariant way as

$$
\begin{aligned}
&\nabla_A e^- = 0\,, \qquad R^{\mathcal{K}}_{AB} = 0\,, \qquad \nabla_X\omega_{ij} = 0\,,\\
&\cos\lambda\,[\nabla_i\omega_{jk}]^{\mathbf{20}+\bar{\mathbf{20}}} + \frac{1}{2}\left[[(d\omega^{2,1} + d\omega^{1,2})\cdot\mathrm{Re}\chi^\lambda]_{ijk}\right]^{\mathbf{20}+\bar{\mathbf{20}}} = 0\,,\\
&d\omega^{3,0} + d\omega^{0,3} = -\frac{1}{2\sin\lambda} i_{d\lambda}\mathrm{Im}\chi^\lambda\\
&\theta_i = \frac{1}{2} d\omega_{jki}\omega^{jk} = -2\frac{1 + 2\sin^2\lambda}{\sin 2\lambda}\partial_i\lambda\,,
\end{aligned}
\tag{3.15}
$$

where $e^- = \frac{1}{\sqrt{2}}(-e^0 + e^5)$, $\chi^\lambda = e^{i\lambda}\chi$, $d\omega^{3,0}$ denotes the (3,0)-part of $d\omega$ with respect to I and similarly for $d\omega^{0,3}$, $d\omega^{2,1}$ and $d\omega^{1,2}$, and $i, j, k = 1, 2, 3, 4, 6, 7, 8, 9$. We have also used the decomposition of tensors in SU(4) representations and

$$
(d\omega^{2,1}\cdot\mathrm{Re}\chi^\lambda)_{ijk} = \frac{1}{2}(d\omega^{2,1})_{imn}(\chi^\lambda)^{mn}{}_{jk}\,.
\tag{3.16}
$$

The condition $R^{\mathcal{K}}_{AB} = 0$ implies that the curvature of the canonical bundle of the spacetime vanishes and this is derived as an integrability condition of $i\partial_A\lambda + \Omega_{A,\alpha}{}^{\alpha} = 0$. The one-form e^- is parallel with respect to the Levi-Civita connection. Therefore the associated vector field $X = e_+$ is null, Killing and rotation free. Adapting coordinates along X, $X = \partial/\partial u$, and using some of the conditions, the metric and three-form H can be written as

$$
\begin{aligned}
ds^2 &= 2e^-e^+ + \delta_{ij}e^ie^j = 2dv(du + Vdv + n) + g_{IJ}dy^Idy^J\,,\\
H &= \frac{1}{2} i_{d\Phi - d_I\lambda}\mathrm{Re}\chi - \cot\lambda[(d\omega)^{1,2} + (d\omega)^{2,1}] - \frac{1}{2\sin\lambda}\left(\frac{1}{3}W_1 + W_2\right)\cdot\mathrm{Re}\chi^\lambda\\
&\quad -e^-\wedge\left[\cot\lambda\,\nabla_-\omega + \frac{1}{2\sin\lambda}\,\nabla_-\omega\cdot\mathrm{Re}\chi^\lambda\right] + \frac{1}{2}e^-\wedge\omega\,\partial_-\lambda + \frac{1}{2}H^{\mathbf{15}}_{-ij}e^-\wedge e^i\wedge e^j\,,\\
e^\Phi &= \ell(v)\cos\lambda\,,
\end{aligned}
\tag{3.17}
$$

where

$$
(W_1\cdot\mathrm{Re}\chi^\lambda)_{\alpha,\bar{\gamma}\bar{\delta}} = \frac{1}{2}(W_1)_{\alpha\beta_1\beta_2}\mathrm{Re}(\chi^\lambda)^{\beta_1\beta_2}{}_{\bar{\gamma}\bar{\delta}}\,,
\tag{3.18}
$$

the classes W_1 and W_2 are defined in appendix C and d_I is the exterior derivative with respect to the endomorphism I. The only component of H that is not determined in terms of the geometry is $H^{\mathbf{su}(4)}_{-ij} = H^{\mathbf{15}}_{-ij}$. The expression for the flux H depends on the trivialization of the canonical bundle λ. All the components of the metric and fluxes are independent of the coordinate u.

The spacetime is a pp-wave propagating in a manifold B with an SU(4) structure given by $v, u = \text{const}$. Alternatively, it can be seen as a Lorentzian two-parameter family of B. The geometry of B can be easily described by restricting the conditions we have presented in (3.15) on B. In particular we have that $\nabla_A \chi^\lambda = 0$ implies that

$$2\tilde{W}_5^\lambda - \tilde{W}_4^\lambda = 0\,, \tag{3.19}$$

where $\tilde{W}_5$ denotes the restriction of the forms that define the class W_5 on B and similarly for the rest. In addition, $\tilde{W}_1$ and $\tilde{W}_4$ are specified in terms of $d\lambda$ as

$$\begin{aligned}\tilde{W}_1 &= -\frac{1}{2\sin\lambda} i_{d\lambda}\mathrm{Im}\chi^\lambda\,,\\ \tilde{W}_4 &= -2\frac{1+2\sin^2\lambda}{\sin 2\lambda} d\lambda\,.\end{aligned} \tag{3.20}$$

Finally, we have

$$2\cos\lambda\, \tilde{W}_2 + \frac{1}{2}[\tilde{W}_3 \cdot \mathrm{Re}\chi^\lambda]^{\mathbf{20}+\bar{\mathbf{20}}} = 0\,. \tag{3.21}$$

Therefore the eight-manifold B is almost hermitian with trivial canonical bundle.

3.3 Backgrounds with G_2-invariant Killing spinors

3.3.1 Supersymmetry conditions

We have seen that the G_2-invariant Killing spinors can be chosen as

$$\hat{\epsilon} = f(1 + e_{1234})\,, \qquad \check{\epsilon} = g(e_{15} + e_{2345})\,. \tag{3.22}$$

The local Spin(9, 1) transformations along the Γ_{+-} direction scale the Killing spinors as $\hat{\epsilon} \to \ell\hat{\epsilon}$ and $\check{\epsilon} \to \ell^{-1}\check{\epsilon}$. Therefore one can choose the gauge $f = g$. In what follows, we shall present our results in this gauge. Of course the geometry of spacetime is independent of the choice of gauge.

The linear system for the above G_2-invariant spinors can be easily constructed from that of the heterotic backgrounds with G_2-invariant Killing spinors [1]. The only difference is that one has to replace H with $-H$ in the conditions that arise from the second Killing spinor $\check{\epsilon}$. Because of this, we shall not give further details. After some computation, the conditions that arise from the gravitino Killing spinor equations can be written as

$$\begin{aligned}&2\partial_A \log f^2 - H_{A-+} = 0\,, \qquad \hat{\nabla}_A(f^2 e^-) = 0\,, \qquad \check{\nabla}_A(f^2 e^+) = 0\,,\\ &H_{A1i} + \frac{1}{12}\nabla_A\varphi_{jkl} \star \varphi^{jkl}{}_i = 0\,, \qquad \nabla_A Z_i - \frac{1}{4}H_{Ajk}\varphi^{jk}{}_i = 0\,,\end{aligned} \tag{3.23}$$

and the conditions that arise from the dilatino Killing spinor equation are

$$\partial_+\Phi = \partial_-\Phi = 0\,, \qquad H_{-1i} + \frac{1}{2}H_{-kl}\varphi^{kl}{}_i = 0\,, \qquad H_{+1i} - \frac{1}{2}H_{+kl}\varphi^{kl}{}_i = 0\,,$$

$$H_{jkl}\star\varphi^{jkl}{}_i=0\,,\qquad 2\partial_1\Phi-\frac{1}{6}H_{ijk}\varphi^{ijk}-H_{-+1}=0\,,$$
$$2\partial_i\Phi+\frac{1}{2}H_{1kl}\varphi^{kl}{}_i-H_{-+i}=0\,, \tag{3.24}$$

where $i,j,k=2,3,4,6,7,8,9$, $\varphi=\mathrm{Re}\,\hat{\chi}+e^6\wedge\hat{\omega}$ is the G_2-invariant three-form, $\hat{\chi}=(e^2+ie^7)\wedge(e^3+ie^8)\wedge(e^4+ie^9)$, $\hat{\omega}=-(e^2\wedge e^7+e^3\wedge e^8+e^4\wedge e^9)$ and $Z=e_1$. The dual $\star\varphi$ of φ is taken with respect to $d\mathrm{vol}=e^2\wedge e^3\wedge e^4\wedge e^6\wedge e^7\wedge e^8\wedge e^9$.

3.3.2 The geometry of spacetime

The spacetime form bilinears of the spinor $\hat{\epsilon}$ have been presented in (3.4). The rest of the form bilinears are

$$\kappa(\hat{\epsilon},\check{\epsilon})=-f^2e^1\,,\qquad \kappa(\check{\epsilon},\check{\epsilon})=f^2(e^0+e^5)\,,\qquad \xi(\hat{\epsilon},\check{\epsilon})=f^2[\mathrm{Re}\hat{\chi}+e^6\wedge\hat{\omega}-e^0\wedge e^1\wedge e^5]\,,$$
$$\tau(\hat{\epsilon},\check{\epsilon})=f^2\left[-\mathrm{Re}\hat{\chi}\wedge e^0\wedge e^5+\mathrm{Im}\hat{\chi}\wedge e^1\wedge e^6+\frac{1}{2}e^1\wedge\hat{\omega}\wedge\hat{\omega}-\hat{\omega}\wedge e^0\wedge e^5\wedge e^6\right],$$
$$\tau(\check{\epsilon},\check{\epsilon})=-f^2(e^0+e^5)\wedge\left[e^1\wedge\mathrm{Re}\hat{\chi}+e^6\wedge\mathrm{Im}\hat{\chi}+\frac{1}{2}\hat{\omega}\wedge\hat{\omega}+\hat{\omega}\wedge e^1\wedge e^6\right]. \tag{3.25}$$

It is clear from (3.23) that $\kappa(\hat{\epsilon},\hat{\epsilon})$ and $\kappa(\check{\epsilon},\check{\epsilon})$ are parallel with respect to the $\hat{\nabla}$ and $\check{\nabla}$ connections, respectively, as may have been expected from the general arguments we have presented in section 2. In fact, a more detailed analysis reveals that $\mathrm{hol}(\hat{\nabla})\subseteq\mathrm{Spin}(7)\ltimes\mathbb{R}^8$ and $\mathrm{hol}(\check{\nabla})\subseteq\mathrm{Spin}(7)\ltimes\mathbb{R}^8$. These holonomy groups are embedded in $\mathrm{Spin}(9,1)$ in different ways. This can also be seen by comparing $\tau(\hat{\epsilon},\hat{\epsilon})$ and $\tau(\check{\epsilon},\check{\epsilon})$.

To solve the conditions (3.23) and (3.24) observe that under G_2 the flux H decomposes as

$$H_{abc}\,,\qquad H_{abi}\,,\qquad H_{aij}\,,\qquad H_{ijk}\,,\qquad a,b,c=+,-,1\,,\qquad i,j,k=2,3,4,6,7,8,9\,. \tag{3.26}$$

In addition the space of two- and three-forms decomposes as $\Lambda^2(\mathbb{R}^7)=\Lambda^2_{\mathbf{7}}\oplus\Lambda^2_{\mathbf{14}}$ and $\Lambda^3(\mathbb{R}^7)=\Lambda^3_{\mathbf{1}}\oplus\Lambda^3_{\mathbf{7}}\oplus\Lambda^3_{\mathbf{27}}$, respectively, where $\Lambda^2_{\mathbf{14}}=\mathfrak{g}_2$. The conditions (3.23) and (3.24) determine all the components of H in terms of the geometry apart from $H^{\mathbf{14}}_{-ij}$ and $H^{\mathbf{14}}_{+ij}$. Let $\hat{X},\check{X}$ and Z be the vector fields associated with the bi-linears f^2e^-, f^2e^+ and e^1, i.e. $\hat{X}=f^2e_+$, $\check{X}=f^2e_-$ and $Z=e_1$. One then finds that

$$2\partial_1\log f^2-H_{1-+}=0\,,\quad H_{-1i}+2\nabla_-Z_i=0\,,\qquad H_{+1i}-2\nabla_+Z_i=0\,,$$
$$2\partial_i\log f^2-H_{i-+}=0\,,\quad H_{1ij}+\frac{1}{12}\nabla_{[i}\varphi^{mnp}\star\varphi_{j]mnp}=0\,,\quad \nabla_+Z_i-\frac{1}{4}H_{+jk}\varphi^{jk}{}_i=0\,,$$
$$\nabla_-Z_i-\frac{1}{4}H_{-jk}\varphi^{jk}{}_i=0\,,\quad H_{ijk}=-\frac{1}{3}\nabla_lZ^l\varphi_{ijk}+3Z_{p[i}\varphi^p{}_{jk]}\,,\quad \partial_-\Phi=\partial_+\Phi=0\,, \tag{3.27}$$

where $Z_{ij}=\nabla_{(i}Z_{j)}$. In addition, the conditions on the geometry are

$$\hat{\nabla}_AX^B=\check{\nabla}_AY^B=0\,,\qquad \nabla_1\varphi_{jkl}\star\varphi^{jkl}{}_i=0\,,\qquad \nabla_{(i}\varphi^{mnp}\star\varphi_{j)mnp}=0\,,$$
$$8\nabla_1Z_i+\nabla^p\varphi_{pmn}\varphi^{mn}{}_i=0\,,\qquad \frac{1}{24}\nabla_-\varphi_{jkl}\star\varphi^{jkl}{}_i+\nabla_-Z_i=0\,,$$
$$\frac{1}{24}\nabla_+\varphi_{jkl}\star\varphi^{jkl}{}_i-\nabla_+Z_i=0\,,\qquad \nabla_{[i}Z_{j]}=0\,,$$

$$\partial_1(\Phi - \log f^2) - \frac{1}{3}\nabla_i Z^i = 0\,, \qquad 2\partial_i(\Phi - \log f^2) - \frac{1}{4}\nabla^p \varphi_{pjk}\varphi^{jk}{}_i = 0\,. \tag{3.28}$$

As we have already mentioned, the conditions on the geometry imply that $\hat{X}, \check{X}$ are parallel with respect to the $\hat{\nabla}$ and $\check{\nabla}$ connections, respectively. This implies that $\hat{X}, \check{X}$ are Killing, commute $[\hat{X}, \check{X}] = 0$ and their rotations are given in terms of H as $d\hat{\kappa}_X = i_{\hat{X}} H$ and $d\check{\kappa}_Y = -i_{\check{Y}} H$, see section 2. In addition, it turns out that $[\hat{X}, Z] = [\check{X}, Z] = 0$ but Z is not Killing. Because of this, it is convenient to consider the spacetime as a fibration over an eight-dimensional space B with fibres given by the orbits of $\hat{X}, \check{X}$. The base space B admits a G_2-structure which we shall specify. Introducing coordinates adapted to the vectors fields $\hat{X}, \check{X}$ and Z, i.e. $\hat{X} = \partial/\partial u$, $\check{X} = \partial/\partial v$ and $Z = \partial/\partial x$, one can write the metric and the three-form as

$$\begin{aligned} ds^2 &= 2f^4(du + n_i e^i)(dv + m_i e^i) + (dx + \ell_i e^i)^2 + \delta_{ij} e^i{}_I e^j{}_J dy^I dy^J \\ H &= d(e^- \wedge e^+) - \frac{1}{24}\nabla_{[i}\varphi^{mnp} \star \varphi_{j]mnp}\, e^1 \wedge e^i \wedge e^j \\ &\quad + [-\frac{1}{18}\nabla_l Z^l \varphi_{ijk} + \frac{1}{2} Z_{p[i}\varphi^p{}_{jk]}]\, e^i \wedge e^j \wedge e^k\,, \\ \Phi &= \Phi(x, y) \end{aligned} \tag{3.29}$$

where

$$e^+ = f^2(du + n_i e^i)\,, \qquad e^- = f^2(dv + m_i e^i)\,, \qquad e^1 = dx + \ell_i e^i\,, \tag{3.30}$$

and all components of the metric and fluxes are independent of u, v. The components m, n of the metric are not arbitrary. In particular, they are related to the rotation of $\hat{X}, \check{X}$ which in turn is related to Z as can be seen by the conditions in (3.27).

It remains to find the conditions that supersymmetry imposes on the eight-dimensional base space B of the fibration. B admits a G_2-structure. The different G_2-structures of an eight-dimensional manifold are described in appendix C. It is clear that the conditions (3.28) imply that

$$\begin{aligned} &\tilde{W} = 0, \qquad \tilde{X}_2 = \tilde{X}_3 = 0\,, \qquad \tilde{W}_4 = 0\,, \qquad 4\tilde{X} + 3\tilde{W}_2 = 0\,, \\ &3\partial_1(\Phi - \log f^2) - \tilde{X}_1 = 0\,, \qquad 4\partial_i(\Phi - \log f^2) - 3\tilde{W}_2 = 0\,, \end{aligned} \tag{3.31}$$

where $\tilde{W}_2$ is represented by the Lee form[5]

$$\tilde{\theta}_i = \frac{1}{6}\nabla^p \varphi_{pmn}\varphi^{mn}{}_i\,, \tag{3.32}$$

$\tilde{W}$ denotes the projection on the base space B of W and similarly for the rest of the classes. The components H_{ijk} are determined by the classes $\tilde{X}_1$ and $\tilde{X}_4$ while the component H_{1ij} is determined by $\tilde{W}_2$ and $\tilde{W}_3$. Observe that if $e^\Phi = f^2$, then $\tilde{W}_2 = \tilde{X} = \tilde{X}_1 = 0$.

[5]In terms of seven-dimensional data $\tilde{\theta} = -\frac{1}{3} \star (\star d\varphi \wedge \varphi)$.

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3.3.3 Field equations

To find the field equations that are implied by the Killing spinor equations, one can use the integrability conditions in appendix A and the results in appendix B. A straightforward calculation reveals that if one imposes

$$dH = 0\,, \qquad LH_{-+} = 0\,, \tag{3.33}$$

then all the rest of the field equations are implied. This is the case for all the $\hat{N} = \check{N} = 1$ supersymmetric common sector backgrounds which admit K-invariant Killing spinors. As the proof is very similar for the other cases we shall not repeat the analysis in each case.

4. IIA $N = 2$ backgrounds

4.1 Backgrounds with Spin(7)-invariant spinors

We have shown in section 2 that the Spin(7)-invariant Killing spinors can be written as

$$\hat{\epsilon} = f(1 + e_{1234})\,, \qquad \check{\epsilon} = g(e_5 + e_{12345})\,. \tag{4.1}$$

Observe that $\hat{\epsilon}$ is an even-degree form while $\check{\epsilon}$ is an odd-degree form. This is because the Killing spinors of the two copies of the IIA Killing spinor equations have opposite chirality. The linear system associated with these Killing spinors can be constructed from that of [1] and the results in appendix B. Under local $\mathrm{Spin}(9,1)$ transformations in the direction Γ_{+-}, the spinors transform as $\hat{\epsilon} \to \ell\hat{\epsilon}$ and $\check{\epsilon} \to \ell^{-1}\check{\epsilon}$. This symmetry can be fixed by setting $f = g$. It turns out that this is a convenient gauge to use for our investigation. The geometry of spacetime does not depend on the choice of gauge.

The linear system associated with the IIA Killing spinor equations for the $\hat{\epsilon}$ and $\check{\epsilon}$ spinors can be solved to give

$$\begin{aligned}
&\Omega_{A,-+} = 0\,, \qquad 2\partial_A \log(f^2) + H_{A+-} = 0\,, \qquad \Omega_{A,\alpha}{}^{\alpha} = 0\,,\\
&\Omega_{A,\bar\alpha\bar\beta} - \frac{1}{2}\Omega_{A,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,, \qquad \hat{\Omega}_{A,+\alpha} = 0\,, \qquad \check{\Omega}_{A,-\alpha} = 0\,,\\
&\partial_+\Phi = 0\,, \qquad H_{+\alpha}{}^{\alpha} = 0\,, \qquad -H_{+\bar\alpha\bar\beta} + \frac{1}{2}H_{+\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&\partial_-\Phi = 0\,, \qquad H_{-\alpha}{}^{\alpha} = 0\,, \qquad -H_{-\bar\alpha\bar\beta} + \frac{1}{2}H_{-\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&H_{ijk} = 0\,, \qquad \partial_{\bar\alpha}\Phi - \frac{1}{2}H_{-+\bar\alpha} = 0\,.
\end{aligned} \tag{4.2}$$

These conditions can be rewritten in a covariant way as

$$\begin{aligned}
&\hat{\nabla}(f^2 e^-) = 0\,, \qquad \check{\nabla}(f^2 e^+) = 0\,, \qquad \nabla_A\phi_{ijkl} = 0\,,\\
&\partial_+\Phi = \partial_-\Phi = 0\,, \qquad \partial_+ f^2 = \partial_- f^2 = 0\,,\\
&-H_{-ij} + \frac{1}{2}H_{-kl}\phi^{kl}{}_{ij} = 0\,, \qquad -H_{+ij} + \frac{1}{2}H_{+kl}\phi^{kl}{}_{ij} = 0\,,\\
&H_{ijk} = 0\,, \qquad \partial_i\Phi - \frac{1}{2}H_{-+i} = 0\,,
\end{aligned} \tag{4.3}$$

where ϕ is the Spin(7)-invariant four-form defined in section 3. It remains to examine the restrictions on the geometry of spacetime imposed by the above conditions.

4.1.1 The geometry of spacetime

The spinor bilinears of $\hat{\epsilon}$ have already been computed in the IIB case. The spinor bi-linears of $\check{\epsilon}$ can be computed from those of $\hat{\epsilon}$ by replacing e^- with e^+. It remains to compute the spacetime forms associated with $(\hat{\epsilon}, \check{\epsilon})$. After an additional normalization of the spinors with $1/\sqrt{2}$, one finds

$$\alpha(\hat{\epsilon}, \check{\epsilon}) = -f^2\,, \qquad \beta(\hat{\epsilon}, \check{\epsilon}) = f^2\, e^0 \wedge e^5\,, \qquad \rho(\hat{\epsilon}, \check{\epsilon}) = -f^2\, \phi\,, \tag{4.4}$$

where ϕ is the Spin(7)-invariant form in section 3. In contrast to the IIB case, the Killing spinors have a non-degenerate inner product.

A consequence of the supersymmetry conditions (4.3) is that the vector fields $\hat{X} = f^2 e_+$ and $\check{X} = f^2 e_-$ are Killing and commute $[\hat{X}, \check{X}] = 0$. Adapting coordinates along $\hat{X}$ and $\check{X}$, $\hat{X} = \partial/\partial u$, $\check{X} = \partial/\partial v$, the metric, torsion and dilaton can be written as

$$\begin{aligned} ds^2 &= 2f^4(dv + m)(du + n) + \delta_{ij} e^i e^j\,, \\ H &= d(e^- \wedge e^+)\,, \\ e^\Phi &= f^2\,, \end{aligned} \tag{4.5}$$

where $e^- = f^2(dv + m)$ and $e^+ = f^2(du + n)$, and all fields are independent of u, v. The components m, n are not arbitrary. In particular, the conditions on H in (4.3) require that dm and dn take values in $\mathfrak{spin}(7)$. Clearly the spacetime is a rank two fibre bundle with fibre given by the orbits of $\hat{X}$ and $\check{X}$ and with base space B a Spin(7) manifold, i.e. $\mathrm{hol}(\tilde{\nabla}) \subseteq \mathrm{Spin}(7)$.

The Bianchi identity for H is automatically satisfied. So, as we have explained in section 3, it remains to impose the field equation $LH_{-+} = 0$. This specifies f. In particular, it implies that f^{-4} is a harmonic function on B. Therefore the spacetime can be thought of as a generalization of a fundamental string of [22] with rotation and wrapping, and with transverse space the Spin(7) manifold B.

4.2 Backgrounds with SU(4)-invariant spinors

It has been shown in section two that the Killing spinors can be chosen as

$$\hat{\epsilon} = f(1 + e_{1234})\,, \qquad \check{\epsilon} = g_1(e_5 + e_{12345}) + i g_2(e_5 - e_{12345})\,. \tag{4.6}$$

The linear system associated with the Killing spinor equations for these spinors can be easily constructed from the results of [1] and those in appendix B. So we shall not give more details.

It is convenient to express the solution of the linear system in the gauge $f = 1$. This is attained with a local Spin(9,1) transformation in the Γ_{+-} direction. After some computation, and setting $g_1 + g_2 = g^2 e^{i\lambda}$, we find that the solution of the linear system can be written as

$$\begin{aligned} &\Omega_{A,\alpha}{}^{\alpha} = \frac{1}{2} H_{A\alpha}{}^{\alpha} = -i\partial_A \lambda\,, \qquad \hat{\Omega}_{A,+B} = 0\,, \qquad \partial_A \log g^2 - \Omega_{A,-+} = 0\,, \qquad \check{\Omega}_{A,-\alpha} = 0\,, \\ &\partial_+ \Phi = \partial_- \Phi = 0\,, \qquad H_{-\alpha}{}^{\alpha} = H_{+\alpha}{}^{\alpha} = 0\,, \qquad e^\Phi = g^2 \cos\lambda\,, \end{aligned}$$

$$
\begin{aligned}
&H_{\beta_1\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = -6\partial_{\bar\alpha}(\log\cos\lambda + i\lambda)\,,\\
&H_{A'\bar\alpha\bar\beta} = \frac{2}{e^{2i\lambda}-1}[-(1+e^{2i\lambda})\Omega_{A',\bar\alpha\bar\beta} + \Omega_{A',\beta_1\beta_2}\epsilon^{\beta_1\beta_2}{}_{\bar\alpha\bar\beta}]\,, \qquad A' = \bar\gamma,\gamma\,.\\
&-(1+e^{2i\lambda})(2\Omega_{\bar\alpha,\bar\beta\bar\gamma} - \Omega_{\bar\gamma,\bar\alpha\bar\beta} - \Omega_{\bar\beta,\bar\gamma\bar\alpha}) + 2\Omega_{\bar\alpha,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\beta\bar\gamma} - \Omega_{\bar\gamma,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\alpha\bar\beta} - \Omega_{\bar\beta,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\gamma\bar\alpha} = 0,\\
&-2\Omega_{+,\bar\alpha\bar\beta} + \Omega_{+,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,, -2\Omega_{-,\bar\alpha\bar\beta} + e^{-2i\lambda}\Omega_{-,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&\Omega_{-,\alpha\beta} - \frac{1}{2}H_{-\alpha\beta} = 0\,, \qquad \Omega_{+,\alpha\beta} + \frac{1}{2}H_{+\alpha\beta} = 0\,.
\end{aligned} \tag{4.7}
$$

In turn, these conditions can be solved to express the fluxes in terms of the geometry

$$
\begin{aligned}
&\frac{1}{2}H_{A\alpha}{}^{\alpha} = -i\partial_A\lambda\,, \qquad H_{-\alpha}{}^{\alpha} = H_{+\alpha}{}^{\alpha} = 0\,, \qquad e^{\Phi} = g^2\cos\lambda\,,\\
&H_{\beta_1\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = -6\partial_{\bar\alpha}(\log\cos\lambda + i\lambda)\,,\\
&H_{\gamma\bar\alpha\bar\beta} = \frac{2}{e^{2i\lambda}-1}[-(1+e^{2i\lambda})\Omega_{\gamma,\bar\alpha\bar\beta} + \Omega_{\gamma,\beta_1\beta_2}\epsilon^{\beta_1\beta_2}{}_{\bar\alpha\bar\beta}]\,,\\
&\Omega_{-,\alpha\beta} - \frac{1}{2}H_{-\alpha\beta} = 0\,, \qquad \Omega_{+\alpha\beta} + \frac{1}{2}H_{+\alpha\beta} = 0\,,
\end{aligned} \tag{4.8}
$$

and to find the conditions

$$
\begin{aligned}
&\hat\Omega_{A,+B} = 0\,, \qquad i\partial_A\lambda + \Omega_{A,\alpha}{}^{\alpha} = 0\,, \qquad \partial_A\log g^2 - \Omega_{A,-+} = 0\,,\\
&\check\Omega_{A,-\alpha} = 0\,, \qquad -2\Omega_{+,\bar\alpha\bar\beta} + \Omega_{+,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,, -2\Omega_{-,\bar\alpha\bar\beta} + e^{-2i\lambda}\Omega_{-,\gamma\delta}\epsilon^{\gamma\delta}{}_{\bar\alpha\bar\beta} = 0\,,\\
&-(1+e^{2i\lambda})(2\Omega_{\bar\alpha,\bar\beta\bar\gamma} - \Omega_{\bar\gamma,\bar\alpha\bar\beta} - \Omega_{\bar\beta,\bar\gamma\bar\alpha}) + 2\Omega_{\bar\alpha,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\beta\bar\gamma} - \Omega_{\bar\gamma,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\alpha\bar\beta} - \Omega_{\bar\beta,\delta_1\delta_2}\epsilon^{\delta_1\delta_2}{}_{\bar\gamma\bar\alpha} = 0,\\
&\Omega_{\beta,}{}^{\beta}{}_{\bar\alpha} = -\frac{1+2\sin^2\lambda}{\sin 2\lambda}\partial_{\bar\alpha}\lambda\,,\\
&\Omega_{\beta_1,\beta_2\beta_3}\epsilon^{\beta_1\beta_2\beta_3}{}_{\bar\alpha} = -\frac{e^{i\lambda}}{\sin\lambda}\partial_{\bar\alpha}\lambda\,,
\end{aligned} \tag{4.9}
$$

on the geometry of spacetime. It is clear that the conditions we have found resemble those of the $SU(4) \ltimes \mathbb{R}^8$ backgrounds of the type IIB common sector.

4.2.1 The geometry of spacetime

The spacetime forms $\hat\alpha$ associated with the spinor $\hat\epsilon$ have been computed in (3.4). Similarly, the spacetime forms of $\check\epsilon$ can be easily constructed from those of the second spinor in the $SU(4) \ltimes \mathbb{R}^8$ case. The remaining spacetime form spinor bilinears are

$$
\begin{aligned}
&\alpha(\hat\epsilon,\check\epsilon) = -g_1\,, \qquad \beta(\hat\epsilon,\check\epsilon) = g_1 e^0\wedge e^5 - g_2\omega\,,\\
&\rho(\hat\epsilon,\check\epsilon) = g_2 e^0\wedge e^5\wedge\omega + \frac{1}{2}g_1\omega\wedge\omega - g_1\mathrm{Re}\chi + g_2\mathrm{Im}\chi\,,
\end{aligned} \tag{4.10}
$$

where ω and χ are defined as in section 3. The conditions on the geometry can now be written in a covariant way as

$$
\begin{aligned}
&\hat\nabla_A e^- = 0\,, \qquad \check\nabla_A(g^2e^+) = 0\,, \qquad R^{\mathcal{K}}_{AB} = 0\,, \qquad \check\nabla_+\omega_{ij} = 0\,, \qquad \hat\nabla_-\omega_{ij} = 0\,,\\
&\nabla_+\phi = 0\,, \qquad \nabla_-\phi^{2\lambda} = 0\,,\\
&\cos\lambda[\nabla_i\omega_{jk}]^{\mathbf{20}+\bar{\mathbf{20}}} + \frac{1}{2}[[(d\omega^{2,1} + d\omega^{1,2})\cdot\mathrm{Re}\chi^{\lambda}]_{ijk}]^{\mathbf{20}+\bar{\mathbf{20}}} = 0\,,\\
&d\omega^{3,0} + d\omega^{0,3} = -\frac{1}{2\sin\lambda}i_{d\lambda}\mathrm{Im}\chi^{\lambda}\\
&\theta_i = \frac{1}{2}d\omega_{jki}\omega^{jk} = -2\frac{1+2\sin^2\lambda}{\sin 2\lambda}\partial_i\lambda\,,
\end{aligned} \tag{4.11}
$$

where $R^{\mathcal{K}}$ is the curvature of the canonical bundle whose condition arises as the integrability condition of $i\partial_A\lambda + \Omega_{A,\alpha}{}^{\alpha} = 0$ and $\phi^{2\lambda} = \mathrm{Re}(e^{2i\lambda}\chi) - \frac{1}{2}\omega\wedge\omega$. The explanation for the rest of the notation can be found in the type IIB $\mathrm{SU}(4)\ltimes\mathbb{R}^8$ case in section 3.

The conditions on the geometry imply that the vector fields $\hat{X} = e_+$ and $\check{X} = g^2 e_-$ are $\hat{\nabla}$- and $\check{\nabla}$- parallel, respectively. Therefore they are Killing and $[\hat{X},\check{X}] = 0$. After adapting coordinates along these Killing vector fields, $\hat{X} = \partial/\partial u$ and $\check{X} = \partial/\partial v$, and after some computation, the spacetime metric torsion and dilaton can be written as

$$\begin{aligned}
&ds^2 = 2f^4(dv+n)(du+m) + \delta_{ij}e^i_I e^j_J dy^I dy^J\\
&H = d(e^-\wedge e^+) + \frac{1}{2}i_{d\Phi - d_I\lambda}\mathrm{Re}\chi - \cot\lambda[(d\omega)^{1,2} + (d\omega)^{2,1}] - \frac{1}{2\sin\lambda}(\frac{1}{3}W_1 + W_2)\cdot\mathrm{Re}\chi^{\lambda}\\
&\Phi = f^4(y)\cos\lambda(y)\,,
\end{aligned}\tag{4.12}$$

where we have set $g = f^2$ and all the fields are independent of the coordinates u, v.

The spacetime is a fibre bundle with fibre given by the orbits of $\hat{X}$ and $\check{X}$ and with base space an eight-dimensional space B. The geometry of B can be easily described using (4.11). It turns out that one finds the *same* conditions as those on the transverse space B of the pp-wave spacetime of $N = 2$ IIB backgrounds with $\mathrm{SU}(4)\ltimes\mathbb{R}^8$-invariant spinors (3.19)-(3.21).

4.3 Backgrounds with $G_2\ltimes\mathbb{R}^8$-invariant spinors

4.3.1 Supersymmetry conditions

As we have demonstrated in section two, the Killing spinors can be chosen[6] as,

$$\hat{\epsilon} = f(1+e_{1234})\,,\qquad \check{\epsilon} = g(e_1 + e_{234})\,.\tag{4.13}$$

The linear system associated with the Killing spinor equations for these spinors can be easily constructed using the results of [1] and those in appendix B. To solve the linear system, it is convenient to work in the gauge $f = 1$ which as we have explained is attained by a local $\mathrm{Spin}(9,1)$ transformation. Then after some computation, the solution of the linear system can be written as

$$\begin{aligned}
&\nabla_A e^- = 0\,,\qquad \frac{1}{12}\nabla_A\varphi_{jkl}\star\varphi^{jkl}{}_i + H_{A1i} = 0\,,\\
&\nabla_A Z_i - \frac{1}{4}H_{Ajk}\varphi^{jk}{}_i = 0\,,\qquad g = f = 1\,,\\
&\partial_+\Phi = 0\,,\qquad H_{jkl}\star\varphi^{jkl}{}_i = 0\,, H_{+AB} = 0\,,\\
&2\partial_1\Phi - \frac{1}{6}H_{jkl}\varphi^{jkl} = 0\,,\qquad 2\partial_i\Phi + \frac{1}{2}H_{1jk}\varphi^{jk}{}_i = 0\,,
\end{aligned}\tag{4.14}$$

where $Z = e_1$. The rest of the notation is described in the G_2 case of section 3 where one can also find the definition of the G_2-invariant three-form φ. It remains to solve these conditions to determine the restrictions on the geometry and fluxes.

[6] Note that $G_2\ltimes\mathbb{R}^8 = (G_2\ltimes\mathbb{R}^7)\times\mathbb{R}$ because $\mathbb{R}^8$ is a reducible representation of G_2.

4.3.2 The geometry of spacetime

The form spinor bilinears of $(\hat{\epsilon}, \hat{\epsilon})$ have already been computed. The form bilinears of $(\check{\epsilon}, \check{\epsilon})$ can be computed from those of $(e_{51} + e_{5234}, e_{51} + e_{5234})$ by replacing e^- with e^+. So the only new form spinor bilinears are those of $(\hat{\epsilon}, \check{\epsilon})$. It is easy to see after an additional normalization of the spinors that

$$\alpha(\hat{\epsilon}, \check{\epsilon}) = 0\,, \qquad \beta(\hat{\epsilon}, \check{\epsilon}) = (e^0 - e^5) \wedge e^1\,, \qquad \rho(\hat{\epsilon}, \check{\epsilon}) = -(e^0 - e^5) \wedge \varphi\,. \tag{4.15}$$

The supersymmetry conditions (4.14) can be solved to express some of the fluxes in terms of the geometry

$$\begin{aligned} &H_{+AB} = 0\,, \quad H_{-1i} = -\frac{1}{12}\nabla_{-}\varphi_{jkl} \star \varphi^{jkl}{}_{i}\,, \quad H^{\mathbf{7}}_{-ij} = \frac{2}{3}\nabla_{-}Z_k\varphi^{k}{}_{ij}\,, \qquad \partial_{+}\Phi = 0\,, \\ &H_{1ij} = -\frac{1}{12}\nabla_{[i}\varphi^{klm} \star \varphi_{j]klm}\,, \qquad H_{ijk} = -\frac{1}{3}\nabla_l Z^l \varphi_{ijk} + 3Z_{p[i}\varphi^{p}{}_{jk]}\,, \end{aligned} \tag{4.16}$$

and to find the conditions

$$\begin{aligned} &\nabla_A e^- = 0\,, \nabla_1\varphi_{jkl} \star \varphi^{jkl}{}_i = 0\,, \qquad \nabla_{+}\varphi_{jkl} \star \varphi^{jkl}{}_i = 0\,, \\ &\nabla_{(i}\varphi^{jkl} \star \varphi_{m)jkl} {=} 0\,, \quad 8\nabla_1 Z_i + \nabla^m\varphi_{mjk}\varphi^{jk}{}_i {=} 0\,, \quad \nabla_{+}Z_i = 0\,, \qquad \nabla_{[i}Z_{j]} = 0\,, \\ &\partial_1\Phi - \frac{1}{3}\nabla_i Z^i = 0\,, \qquad 2\partial_i\Phi - \frac{1}{4}\nabla^l\varphi_{ljk}\varphi^{jk}{}_i = 0\,, \end{aligned} \tag{4.17}$$

on the geometry of spacetime. It is clear from the above geometric conditions that the null vector field $\hat{X} = e_+$ is parallel with respect to the Levi-Civita connection. This implies that $\hat{X}$ is Killing and rotation free. Adapting coordinates along $\hat{X} = \partial/\partial u$, the metric and torsion can be written as

$$\begin{aligned} ds^2 &= 2dv(du + Vdv + n) + \delta_{ij}e^i_I e^j_J dy^I dy^J \\ H &= -\frac{1}{12}\nabla_{-}\varphi_{jkl} \star \varphi^{jkl}{}_i\, e^- \wedge e^1 \wedge e^i + \frac{1}{3}\nabla_{-}Z_k\varphi^k{}_{ij}\, e^- \wedge e^i \wedge e^j \\ &\quad + \frac{1}{2}H^{\mathbf{14}}_{-ij}\, e^- \wedge e^i \wedge e^j - \frac{1}{24}\nabla_{[i}\varphi^{klm} \star \varphi_{j]klm}\, e^1 \wedge e^i \wedge e^j \\ &\quad + \left[-\frac{1}{18}\nabla_l Z^l\varphi_{ijk} + \frac{1}{2}Z_{p[i}\varphi^p{}_{jk]}\right] e^i \wedge e^j \wedge e^k\,, \\ \Phi &= \Phi(v, y)\,, \end{aligned} \tag{4.18}$$

where $e^- = dv$ and $e^+ = (du{+}Vdv{+}n)$, and all the fields are independent of the coordinate u. Therefore the only component of H that is not determined in terms of the geometry is $H^{\mathbf{14}}_{-ij}$.

The spacetime is a pp-wave propagating in the transverse space B defined by $u, v =$ const. Alternatively, it can be interpreted as a Lorentzian deformation family of B. The manifold B is an eight-dimensional manifold equipped with a G_2-structure. The geometry of such manifolds has been described in appendix C. It is straightforward to find the conditions that supersymmetry imposes on the G_2-structure of B. In particular, one finds that

$$\begin{aligned} &\tilde{W} = 0\,, \qquad \tilde{X}_2 = \tilde{X}_3 = 0\,, \qquad \tilde{W}_4 = 0\,, \qquad 4\tilde{X} + 3\tilde{W}_2 = 0\,, \\ &3\partial_1\tilde{\Phi} - \tilde{X}_1 = 0\,, \qquad 4\partial_i\tilde{\Phi} - 3\tilde{W}_2 = 0\,, \end{aligned} \tag{4.19}$$

where $\tilde{W}$ denotes the restriction of W on B and similarly for the rest of the classes. So up to a redefinition of the dilaton, B has the *same* geometry as the base space of the fibration that arises in the $N = 2$ supersymmetric backgrounds with G_2-invariant spinors in the type IIB common sector (3.31).

5. Type II strings in $N = 2$ backgrounds

5.1 Light-cone gauge fixing

As an application of our results, we shall show that one can always gauge fix the world-volume action of a string propagating in $\hat{N}, \check{N} \geq 1$ supersymmetric backgrounds in the light-cone gauge. Since the $\hat{N}, \check{N} \geq 1$ supersymmetric backgrounds are special cases of the $\hat{N} = \check{N} = 1$ ones, it suffices to demonstrate this for the latter. The details of the gauge fixing procedure depend on whether the stability subgroup of the Killing spinors is K or $K \ltimes \mathbb{R}^8$. First let us consider the latter case. After putting the world-sheet metric in conformal gauge, the bosonic part of the Lagrangian of a string propagating in background (1.2) is

$$\begin{aligned} L = {} & \partial_{\ddagger} v(\partial_{=} u + V\partial_{=} v + (n_I + b_I)\partial_{=} y^I) + \partial_{=} v(\partial_{\ddagger} u + V\partial_{\ddagger} v + (n_I - b_I)\partial_{\ddagger} y^I) \\ & +(g_{IJ} + b_{IJ})\partial_{\ddagger} y^I \partial_{=} y^J \,, \end{aligned} \tag{5.1}$$

where the partial derivatives $\partial_{=}, \partial_{\ddagger}$ have been taken with respect to the world-volume light-cone coordinates $\sigma^{=}, \sigma^{\ddagger}$ of the string and we have identified the embedding map of the string with the spacetime coordinates. It is well-known that this action is invariant under the conformal transformations $\delta y^M = \alpha(\sigma^{\ddagger})\partial_{\ddagger} y^M + \beta(\sigma^{=})\partial_{=} y^M$, where α, β are infinitesimal parameters. It remains to fix this residual symmetry in the light-cone gauge, see e.g. [23]. For this, observe that the equation of motion for u implies that $\partial_{\ddagger}\partial_{=} v = 0$, i.e. v is a free boson. Then, one can choose as the light-cone gauge condition $v = p_{\ddagger}\sigma^{\ddagger} + p_{=}\sigma^{=}$, where $p_{\ddagger}, p_{=}$ are constants[7]. The light-cone Lagrangian reads

$$L_{\text{l.c.}} = (g_{IJ} + b_{IJ})\partial_{\ddagger} y^I \partial_{=} y^J + 2p_{\ddagger}p_{=}V + p_{\ddagger}(n_I + b_I)\partial_{=} y^I + p_{=}(n_I - b_I)\partial_{\ddagger} y^I \,, \tag{5.2}$$

where $H = db$ and $b = b_i e^- \wedge e^i + \frac{1}{2} b_{ij} e^i \wedge e^j$. As usual, the u component of the embedding is determined by the vanishing of the two-dimensional energy-momentum tensor $T_{\ddagger\ddagger} = T_{==} = 0$ in terms of y^I.

The light-cone gauge fixing of strings propagating in the background (1.3) is somewhat different. After writing the world-sheet metric in conformal gauge, the bosonic part of the Lagrangian of a string propagating in the background (1.3) is

$$L = 2f^4 \left(\partial_{\ddagger} v + m_I \partial_{\ddagger} y^I\right) \left(\partial_{=} u + n_I \partial_{=} y^I\right) + \left(g_{IJ} + b_{IJ}\right) \partial_{\ddagger} y^I \partial_{=} y^J \,. \tag{5.3}$$

The equations of motion for u, v imply that

$$\partial_{=} \left[f^4(\partial_{\ddagger} v + m_I \partial_{\ddagger} y^I)\right] = 0 \,, \qquad \partial_{\ddagger} \left[f^4(\partial_{=} u + n_I \partial_{=} y^I)\right] = 0 \,. \tag{5.4}$$

[7] If the string does not wrap around a circle, one takes $p_{\ddagger} = p_{=} = \frac{p}{2}$.

Therefore, the theory has two chiral $U(1)$ currents $\hat{J}_{\ddagger} = f^4(\partial_{\ddagger}v + m_I\partial_{\ddagger}y^I)$ and $\check{J}_{=} = f^4(\partial_{=}u + n_I\partial_{=}y^I)$. The light-cone gauge fixing condition is chosen as

$$\hat{J}_{\ddagger} = \frac{p_{\ddagger}}{2}\,, \qquad \check{J}_{=} = \frac{p_{=}}{2}\,, \tag{5.5}$$

where $p_{\ddagger}, p_{=}$ are constants. The energy momentum conditions $T_{\ddagger\ddagger} = T_{==} = 0$ imply that

$$\begin{aligned} &p_{\ddagger}(\partial_{\ddagger}u + n_I\partial_{\ddagger}y^I) + \gamma_{IJ}\partial_{\ddagger}y^I\partial_{\ddagger}y^J = 0\,, \\ &p_{=}(\partial_{=}v + m_I\partial_{=}y^I) + \gamma_{IJ}\partial_{=}y^I\partial_{=}y^J = 0\,. \end{aligned} \tag{5.6}$$

Therefore, the light-cone gauge fixing conditions together with $T_{\ddagger\ddagger} = T_{==} = 0$ determine u, v in terms of y. Finally, the light-cone action of y is

$$L_{\text{l.c.}} = (g_{IJ} + b_{IJ})\partial_{\ddagger}y^I\partial_{=}y^J - \frac{1}{2}p_{=}p_{\ddagger}f^{-4} + p_{=}m_I\partial_{\ddagger}y^I + p_{\ddagger}n_I\partial_{=}y^I\,. \tag{5.7}$$

Observe the similarity of the light-cone actions (5.2) and (5.7). This similarity is due to the T-duality between these type II backgrounds.

To summarize, the conformal symmetry of the world-sheet action of the string can be fixed in the light-cone gauge for all backgrounds with $\hat{N}, \check{N} \geq 1$ supersymmetry. This does not extend to generic $N \geq 2$ backgrounds with either $\hat{N} = 0$ or $\check{N} = 0$. A similar argument to the one we have used above reveals that for those backgrounds only part of the conformal symmetry of the string world-sheet action can be fixed in the light-cone gauge, see [1].

5.2 Spacetime supersymmetries and world-sheet W-symmetries

One of the questions that arises is the relation between the symmetries of the string world-sheet action and the spacetime supersymmetry of the background in which the string propagates. First, let us focus on the relation between world-sheet and spacetime supersymmetry. As we have seen the eight-dimensional manifold B associated with $\hat{N} = \check{N} = 1$ supersymmetric backgrounds has either Spin(7), SU(4) or G_2 geometry. At first sight this may suggest that the world-sheet action could admit (1,1) supersymmetry which might enhance in the SU(4) case to at least (2,1) or (1,2). Indeed before light-cone gauge fixing, it is straightforward to write the world-volume string action in terms of (1,1) superfields, see e.g. [23] and references therein. Moreover, there is a W-type of conserved current for every $\hat{\nabla}$- or $\check{\nabla}$-parallel form constructed from the Killing spinor bilinears $\hat{\alpha}$ and $\check{\alpha}$ [24]. Choosing the (super)conformal gauge for the world-sheet geometry, the string Lagrangian can be written as

$$L = (g + b)_{MN}D_+Y^MD_-Y^N \tag{5.8}$$

in terms of (1,1) superfields Y. The currents are

$$\hat{J} = \hat{\alpha}_{M_1\dots M_k}\,D_+Y^{M_1}\dots D_+Y^{M_k}\,, \qquad \check{J} = \check{\alpha}_{M_1\dots M_k}\,D_-Y^{M_1}\dots D_-Y^{M_k}\,, \tag{5.9}$$

and are conserved, $D_-\hat{J} = 0$ and $D_+\check{J} = 0$, subject to field equations, where D_+ and D_- are superspace derivatives, and $D_-^2 = i\partial_=$ and $D_+^2 = i\partial_{\ddagger}$.

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The bilinears α constructed from one $\hat{\nabla}$- and one $\check{\nabla}$-parallel spinor are not associated with conserved world-sheet currents even though, as we have seen, they are instrumental in understanding the geometry of the supersymmetric supergravity backgrounds. Applying this to the backgrounds with either SU(4)- or SU(4) $\ltimes \mathbb{R}^8$-invariant spinors, one concludes that the world-sheet supersymmetry does not enhance to either (2,1) or (1,2). This is because there is not a conserved current associated with the additional supersymmetry. Therefore this geometry is different from that found in the context of two-dimensional sigma models in [25].

After light-cone gauge fixing, the world-sheet supersymmetry of the light-cone action is not apparent. For example, the world-sheet light-cone action for the pp-wave type of backgrounds for which $\partial/\partial v$ is not Killing has an explicit dependence on the world-volume coordinates $\sigma^=, \sigma^{\ddagger}$. Therefore it cannot be supersymmetric. In addition, the light-cone actions (5.2) and (5.7) have scalar potential terms. To see whether these are compatible with world-sheet supersymmetry, first observe that B does not necessarily admit a Killing vector field and then compare the scalar potential terms with those of (1,1)-supersymmetric sigma models with torsion found in [26]. Compatibility would require that there was a function h such that $|dh|^2 = U$, where $U = -2p_{\ddagger}p_= V$ or $U = \frac{1}{2}p_{\ddagger}p_= f^{-4}$. So supersymmetrization of the world-sheet action depends on the existence of h.

6. Concluding remarks

We have determined the geometry of all type II common sector $N = 2$ backgrounds. In particular, we have shown that the stability subgroups of the Killing spinors in $\mathrm{Spin}(9,1)$ for the type IIA backgrounds are SU(4) $\ltimes \mathbb{R}^8$ ($\hat{N} = 2, \check{N} = 0$), G_2 ($\hat{N} = 2, \check{N} = 0$), Spin(7) ($\hat{N} = 1, \check{N} = 1$), SU(4) ($\hat{N} = 1, \check{N} = 1$) and $G_2 \ltimes \mathbb{R}^8$ ($\hat{N} = 1, \check{N} = 1$), where $\hat{N}$ and $\check{N}$ denote the positive and negative chirality Killing spinors, respectively. The backgrounds with $\check{N} = 0$ are embeddings of $N = 2$ backgrounds of the heterotic string and their geometries have been analyzed in [1]. The spacetime of backgrounds with Spin(7)- and SU(4)-invariant Killing spinors is a fibre bundle with fibre directions given by the orbits of two null commuting Killing vectors and with base space B an eight-dimensional manifold with a Spin(7)- and an SU(4)-structure, respectively. In particular, in the SU(4) case the almost complex structure of B is not integrable and the fluxes depend on the trivialization of the canonical bundle. The spacetime of backgrounds with $G_2 \ltimes \mathbb{R}^8$-invariant Killing spinors is a pp-wave propagating in an eight-dimensional manifold with a G_2-structure. Alternatively, the spacetime can be viewed as a Lorentzian deformation family of an eight-dimensional manifold with a G_2-structure.

Similarly, we have shown that the stability subgroups of the Killing spinors in $\mathrm{Spin}(9,1)$ of the type IIB backgrounds are SU(4) $\ltimes \mathbb{R}^8$ ($\hat{N} = 2, \check{N} = 0$), G_2 ($\hat{N} = 2, \check{N} = 0$), Spin(7) $\ltimes \mathbb{R}^8$ ($\hat{N} = 1, \check{N} = 1$), SU(4) $\ltimes \mathbb{R}^8$ ($\hat{N} = 1, \check{N} = 1$) and G_2 ($\hat{N} = 1, \check{N} = 1$). As in the IIA case, the backgrounds with $\check{N} = 0$ are embeddings of $N = 2$ backgrounds of the heterotic string and their geometries have been analyzed in [1]. The spacetime of backgrounds with Spin(7) $\ltimes \mathbb{R}^8$- and SU(4) $\ltimes \mathbb{R}^8$-invariant Killing spinors is a pp-wave propagating in a Spin(7) manifold and in an almost hermitian manifold which admits an

SU(4)-structure, respectively. The spacetime of backgrounds with G_2-invariant spinors is a fibre bundle with fibre given by the orbits of two commuting null Killing vectors and with base space an eight-dimensional manifold with a G_2-structure.

We have shown that there is a correspondence between type IIA and IIB common sector backgrounds with $\hat{N} = \check{N} = 1$ supersymmetry. In particular, to every type IIA background with a spacetime geometry that has an interpretation as a Lorentzian family of an eight-dimensional manifold with a K-structure there corresponds a type IIB background with spacetime geometry that of a rank two fibre bundle, and vice versa. The geometry of the deformed manifold and that of the base space of the principal bundle are the *same*. The structure groups of the spacetimes are interchanged as

$$K \leftrightarrow K \ltimes \mathbb{R}^8 \tag{6.1}$$

under this correspondence, where $K = \mathrm{Spin}(7), \mathrm{SU}(4)$ and G_2. This correspondence may have been expected because of the type II T-duality [27 − 30]. It is known that the T-dual background of a fibre bundle along a spacelike fibre isometry direction is a trivial Lorentzian family, i.e. it is a pp-wave background with two commuting isometries generated by the vector fields $\partial/\partial v$ and $\partial/\partial u$. As we have seen the correspondence persists after localization in the v coordinate for the Lorentzian family because it does not change the geometry of the deformed eight-dimensional manifold.

The question arises whether the geometries of common sector backgrounds with $N > 2$ supersymmetries can be classified as we have done here for the $N = 2$ backgrounds. The investigation of backgrounds with either $\hat{N} = 0$ or $\check{N} = 0$ reduces to that of the heterotic string and therefore the result can be found in [1]. A preliminary investigation for the remaining cases, $\hat{N}, \check{N} > 0$ and $N = \hat{N} + \check{N} > 2$, has revealed that more than one hundred new geometries can occur. This is because there are many ways to embed the stability subgroups $\hat{G}$ and $\check{G}$ of the Killing spinors in Spin(9, 1). This leads to many different stability subgroups $G = \hat{G} \cap \check{G}$ of all Killing spinors in Spin(9, 1) for the same number of supersymmetries N. Nevertheless, all these geometries can be classified using the techniques we have employed in this paper and in [1]. The completion of the programme will give an understanding of the geometries of type II common sector backgrounds with any number of supersymmetries.

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A. Common sector field and Killing spinor equations

The Killing spinor equations of the common sector of type II supergravities in components are

$$\hat{\nabla}\hat{\epsilon} = 0\,, \qquad \left(\Gamma^M \partial_M \Phi - \frac{1}{12}\Gamma^{MNP} H_{MNP}\right)\hat{\epsilon} = 0\,,$$

$$\check{\nabla}\check{\epsilon} = 0\,, \qquad \left(\Gamma^M\partial_M\Phi + \frac{1}{12}\Gamma^{MNP}H_{MNP}\right)\check{\epsilon} = 0\,, \tag{A.1}$$

where

$$\hat{\nabla}_N Y^M = \nabla_N Y^M + \frac{1}{2}H^M{}_{NR}Y^R\,, \tag{A.2}$$

$\nabla_M\epsilon = \partial_M\epsilon + \frac{1}{4}\Omega_{M,AB}\Gamma^{AB}\epsilon$, and similarly for $\check{\nabla}$ after setting $H \to -H$, and $\hat{\epsilon}$ and $\check{\epsilon}$ are Majorana-Weyl spinors of the same (IIB) or opposite (IIA) chiralities. The integrability conditions of the Killing spinor equations are

$$\begin{aligned}
-2E_{MN}\Gamma^N\hat{\epsilon} - e^{2\Phi}LH_{MN}\Gamma^N\hat{\epsilon} &= 0\,,\\
L\Phi\hat{\epsilon} - \frac{1}{4}e^{2\Phi}LH_{MN}\Gamma^{MN}\hat{\epsilon} &= 0\,,\\
-2E_{MN}\Gamma^N\check{\epsilon} + e^{2\Phi}LH_{MN}\Gamma^N\check{\epsilon} &= 0\,,\\
L\Phi\check{\epsilon} + \frac{1}{4}e^{2\Phi}LH_{MN}\Gamma^{MN}\check{\epsilon} &= 0\,,
\end{aligned} \tag{A.3}$$

where the field equations in the string frame to lowest order in α' are

$$\begin{aligned}
E_{MN} = R_{MN} - \frac{1}{4}H_{PQM}H^{PQ}{}_N + 2\nabla_M\partial_N\Phi &= 0\,,\\
LH_{PQ} = \nabla_M(e^{-2\Phi}H^M{}_{PQ}) &= 0\,,\\
L\Phi = \nabla^2\Phi - 2g^{MN}\partial_M\Phi\partial_N\Phi + \frac{1}{12}H_{MNR}H^{MNR} &= 0\,.
\end{aligned} \tag{A.4}$$

These integrability conditions are easily derived from those of the heterotic string, see [31, 1]. We have also imposed the Bianchi identity of H, $dH = 0$.

B. Linear system

To construct the linear systems associated with the Killing spinor equations, one has to evaluate the supercovariant derivative and the algebraic Killing spinor equations on a basis in the space of spinors, for details see [21]. For the type IIB common sector, this calculation can be read off from that of the heterotic string in [1]. Similarly, for the type IIA common sector, the evaluation of the $\hat{\nabla}$ and $d\Phi - \frac{1}{2}H$ Killing spinor equations is identical to that of the heterotic string [1]. It remains to evaluate $\check{\nabla}$ and $d\Phi + \frac{1}{2}H$ on a basis of the negative chirality spinors $\Delta^-_{\mathbf{16}}$ spanned by forms of odd degree. A basis in the space of these spinors is $e_5, e_i, e_{ijk}, e_{ij5}$ and e_{12345}. The construction of the spinor representations and the spinor conventions can be found in [1]. In particular, one finds:

$$\begin{aligned}
\frac{1}{4}\check{\Omega}_{A,BC}\Gamma^{BC}e_5 = {}& \frac{1}{2\sqrt{2}}\left(\check{\Omega}_{A,+-} + \check{\Omega}_{A,k}{}^k\right)\Gamma^+1 - \frac{1}{\sqrt{2}}\check{\Omega}_{A,-\bar{k}}\Gamma^{\bar{k}}1\\
&+\frac{1}{4\sqrt{2}}\check{\Omega}_{A,\bar{k}\bar{l}}\Gamma^+\Gamma^{\bar{k}\bar{l}}1\,,
\end{aligned} \tag{B.1}$$

$$\frac{1}{4}\check{\Omega}_{A,BC}\Gamma^{BC}e_{12345} = \frac{1}{2\sqrt{2}}\left(\check{\Omega}_{A,+-} - \check{\Omega}_{A,k}{}^k\right)\Gamma^+e_{1234} - \frac{1}{8\sqrt{2}}\check{\Omega}_{A,kl}\epsilon^{kl}{}_{\bar{m}\bar{n}}\Gamma^+\Gamma^{\bar{m}\bar{n}}1$$

$$-\frac{1}{12\sqrt{2}}\check{\Omega}_{A,-k}\epsilon^{k}{}_{\bar{l}\bar{m}\bar{n}}\Gamma^{\bar{l}\bar{m}\bar{n}}1\,, \tag{B.2}$$

$$\begin{aligned}\frac{1}{4}\check{\Omega}_{A,BC}\Gamma^{BC}e_{\alpha} \,=\, &\frac{1}{2\sqrt{2}}\left(-\check{\Omega}_{A,+-}+\check{\Omega}_{A,\bar{\alpha}\alpha}+\check{\Omega}_{A,k}{}^{k}\right)\Gamma^{\bar{\alpha}}1+\frac{1}{\sqrt{2}}\check{\Omega}_{A,+\alpha}\Gamma^{+}1\\ &+\frac{1}{2\sqrt{2}}\check{\Omega}_{A,+\bar{k}}\Gamma^{+}\Gamma^{\bar{k}}\Gamma^{\bar{\alpha}}1+\frac{1}{4\sqrt{2}}\check{\Omega}_{A,\bar{k}\bar{l}}\Gamma^{\bar{\alpha}}\Gamma^{\bar{k}\bar{l}}1+\frac{1}{\sqrt{2}}\check{\Omega}_{A,\bar{k}\alpha}\Gamma^{\bar{k}}1\,,\end{aligned} \tag{B.3}$$

$$\begin{aligned}\frac{1}{4}\check{\Omega}_{A,BC}\Gamma^{BC}\Gamma^{\alpha}e_{1234} \,=\, &\check{\Omega}_{A,+\bar{\alpha}}\Gamma^{+}e_{1234}-\frac{1}{4}\check{\Omega}_{A,\bar{\alpha}k}\epsilon^{k}{}_{\bar{\alpha}\bar{l}\bar{m}}\Gamma^{\bar{\alpha}\bar{l}\bar{m}}1\\ &+\frac{1}{24}\left(-\check{\Omega}_{A,+-}-\check{\Omega}_{A,\bar{\alpha}\alpha}-\check{\Omega}_{A,l}{}^{l}\right)\epsilon^{\alpha}{}_{\bar{k}\bar{l}\bar{m}}\Gamma^{\bar{k}\bar{l}\bar{m}}1\\ &+\frac{1}{4}\check{\Omega}_{A,+k}\epsilon^{\alpha k}{}_{\bar{l}\bar{m}}\Gamma^{+}\Gamma^{\bar{l}\bar{m}}1-\frac{1}{2}\check{\Omega}_{A,kl}\epsilon^{\alpha kl}{}_{\bar{m}}\Gamma^{\bar{m}}1\,,\end{aligned} \tag{B.4}$$

$$\begin{aligned}\frac{1}{4}\check{\Omega}_{A,BC}\Gamma^{BC}\Gamma^{+}e_{\alpha\beta} \,=\, &-\check{\Omega}_{A,\alpha\beta}\Gamma^{+}1+\frac{1}{2}\check{\Omega}_{A,\bar{k}\bar{l}}\epsilon^{\bar{k}\bar{l}}{}_{\alpha\beta}\Gamma^{+}e_{1234}\\ &+\frac{1}{4}\left(\check{\Omega}_{A,+-}-\check{\Omega}_{A,\gamma}{}^{\gamma}+\check{\Omega}_{A,k}{}^{k}\right)\Gamma^{+}\Gamma_{\alpha\beta}1-\frac{1}{2}\check{\Omega}_{A,-\bar{k}}\Gamma_{\alpha\beta}\Gamma^{\bar{k}}1\\ &-2\check{\Omega}_{A,-[\alpha}\Gamma_{\beta]}1-\check{\Omega}_{A,\bar{k}[\alpha}\Gamma^{+}\Gamma_{\beta]}\Gamma^{\bar{k}}1\,,\end{aligned} \tag{B.5}$$

$$\begin{aligned}\left(d\Phi+\frac{1}{2}H\right)e_{5} \,=\, &\sqrt{2}\left(\partial_{-}\Phi+\frac{1}{2}H_{-k}{}^{k}\right)1-\frac{1}{\sqrt{2}}\left(\partial_{\bar{k}}\Phi+\frac{1}{2}H_{+-\bar{k}}+\frac{1}{2}H_{\bar{k}l}{}^{l}\right)\Gamma^{+}\Gamma^{\bar{k}}1\\ &+\frac{1}{2\sqrt{2}}H_{-\bar{k}\bar{l}}\Gamma^{\bar{k}\bar{l}}1-\frac{1}{12\sqrt{2}}H_{\bar{k}\bar{l}\bar{m}}\Gamma^{+}\Gamma^{\bar{k}\bar{l}\bar{m}}1\,,\end{aligned} \tag{B.6}$$

$$\begin{aligned}\left(d\Phi+\frac{1}{2}H\right)e_{12345} \,=\, &\sqrt{2}\left(\partial_{-}\Phi-\frac{1}{2}H_{-k}{}^{k}\right)e_{1234}\\ &-\frac{1}{12\sqrt{2}}\left(\partial_{k}\Phi+\frac{1}{2}H_{+-k}-\frac{1}{2}H_{kl}{}^{l}\right)\epsilon^{k}{}_{\bar{m}\bar{n}\bar{p}}\Gamma^{+}\Gamma^{\bar{m}\bar{n}\bar{p}}1\\ &-\frac{1}{4\sqrt{2}}H_{-kl}\epsilon^{kl}{}_{\bar{m}\bar{n}}\Gamma^{\bar{m}\bar{n}}1+\frac{1}{6\sqrt{2}}H_{klm}\epsilon^{klm}{}_{\bar{n}}\Gamma^{+}\Gamma^{\bar{n}}1\,,\end{aligned} \tag{B.7}$$

$$\begin{aligned}\left(d\Phi+\frac{1}{2}H\right)e_{\alpha} \,=\, &\sqrt{2}\left(\partial_{\alpha}\Phi-\frac{1}{2}H_{+-\alpha}+\frac{1}{2}H_{\alpha k}{}^{k}\right)1\\ &-\frac{1}{\sqrt{2}}H_{+\alpha\bar{k}}\Gamma^{+}\Gamma^{\bar{k}}1-\frac{1}{3\sqrt{2}}H_{\bar{k}\bar{l}\bar{m}}\epsilon^{\bar{\alpha}\bar{k}\bar{l}\bar{m}}e_{1234}\\ &+\frac{1}{\sqrt{2}}\left(\partial_{+}\Phi+\frac{1}{2}H_{+\bar{\alpha}\alpha}+\frac{1}{2}H_{+k}{}^{k}\right)\Gamma^{+}\Gamma^{\bar{\alpha}}1\\ &+\frac{1}{\sqrt{2}}\left(-\partial_{\bar{k}}\Phi+\frac{1}{2}H_{+-\bar{k}}+\frac{1}{2}H_{\alpha\bar{\alpha}\bar{k}}-\frac{1}{2}H_{\bar{k}l}{}^{l}\right)\Gamma^{\bar{\alpha}}\Gamma^{\bar{k}}1\\ &+\frac{1}{4\sqrt{2}}H_{+\bar{k}\bar{l}}\Gamma^{+}\Gamma^{\bar{\alpha}}\Gamma^{\bar{k}\bar{l}}1+\frac{1}{2\sqrt{2}}H_{\alpha\bar{k}\bar{l}}\Gamma^{\bar{k}\bar{l}}1\,,\end{aligned} \tag{B.8}$$

$$\left(d\Phi+\frac{1}{2}H\right)\Gamma^{\alpha}e_{1234} \,=\, 2\left(\partial_{\bar{\alpha}}\Phi-\frac{1}{2}H_{\bar{\alpha}k}{}^{k}-\frac{1}{2}H_{+-\bar{\alpha}}\right)e_{1234}$$

$$
\begin{aligned}
&+\frac{1}{12}\left(\partial_+\Phi-\frac{1}{2}H_{+\bar\alpha\alpha}-\frac{1}{2}H_{+k}{}^k\right)\epsilon^{\alpha}{}_{\bar l\bar m\bar n}\Gamma^+\Gamma^{\bar l\bar m\bar n}1\\
&+\frac{1}{2}\left(\partial_k\Phi-\frac{1}{2}H_{+-k}-\frac{1}{2}H_{\bar\alpha\alpha k}-\frac{1}{2}H_{kl}{}^l\right)\epsilon^{\alpha k}{}_{\bar l\bar m}\Gamma^{\bar l\bar m}1\\
&-\frac{1}{2}H_{+kl}\epsilon^{\alpha kl}{}_{\bar m}\Gamma^+\Gamma^{\bar m}1+\frac{1}{4}H_{+\bar\alpha k}\epsilon^{\alpha k}{}_{\bar l\bar m}\Gamma^+\Gamma^{\bar\alpha}\Gamma^{\bar l\bar m}1\\
&-\frac{1}{3}H_{klm}\epsilon^{\alpha klm}1-\frac{1}{2}H_{\bar\alpha kl}\epsilon^{\alpha kl}{}_{\bar m}\Gamma^{\bar\alpha}\Gamma^{\bar m}1\,, \qquad \text{(B.9)}
\end{aligned}
$$

$$
\begin{aligned}
\left(d\Phi+\frac{1}{2}H\right)\Gamma^+e_{\alpha\beta}&=-2H_{-\alpha\beta}+H_{-\bar k\bar l}\epsilon_{\alpha\beta}{}^{\bar k\bar l}e_{1234}\\
&+\left(\partial_-\Phi-\frac{1}{2}H_{-\gamma}{}^{\gamma}+\frac{1}{2}H_{-k}{}^k\right)\Gamma_{\alpha\beta}1\\
&+2\left(-\partial_{[\alpha}\Phi-\frac{1}{2}H_{+-[\alpha}+\frac{1}{2}H_{\gamma}{}^{\gamma}{}_{[\alpha}-\frac{1}{2}H_k{}^k{}_{[\alpha}\right)\Gamma^+\Gamma_{\beta]}1\\
&+\frac{1}{2}\left(-\partial_{\bar k}\Phi-\frac{1}{2}H_{+-\bar k}+\frac{1}{2}H_{\bar k\gamma}{}^{\gamma}-\frac{1}{2}H_{\bar kl}{}^l\right)\Gamma^+\Gamma_{\alpha\beta}\Gamma^{\bar k}1\\
&+H_{\bar k\alpha\beta}\Gamma^+\Gamma^{\bar k}1-2H_{-\bar k[\alpha}\Gamma_{\beta]}\Gamma^{\bar k}1-\frac{1}{2}H_{\bar k\bar l[\alpha}\Gamma^+\Gamma_{\beta]}\Gamma^{\bar k\bar l}1\,, \qquad \text{(B.10)}
\end{aligned}
$$

where $A,B,C=\{+,-,\alpha,\bar\alpha,k,\bar k\}$, the range of the Greek indices determined by the number of Greek indices on the left hand side of the respective expression and the range of the Latin indices given by $\mathrm{Range}(\alpha)\cup\mathrm{Range(k)}=\{1,2,3,4\}$. In addition $d\Phi+\frac{1}{2}H=\Gamma^A\partial_A\phi+\frac{1}{12}H_{ABC}\Gamma^{ABC}$.

C. SU(4) and G_2 geometries in eight dimensions

C.1 SU(4)-structures in eight dimensions

SU(4) geometries on an eight-dimensional manifold are characterized by the existence of an almost complex structure I compatible with a Riemannian metric g and a $(4,0)$-form χ such that

$$
\frac{1}{4!}\omega\wedge\omega\wedge\omega\wedge\omega=\frac{1}{2^4}\,\chi\wedge\bar\chi=d\mathrm{vol}\,,\qquad \omega\wedge\chi=0\,, \qquad \text{(C.1)}
$$

where ω is the Hermitian, or Kähler, two-form. The intrinsic torsion of such a manifold decomposes in terms of five irreducible SU(4) representations and therefore there are 2^5 SU(4)-structures in an eight-dimensional manifold, see also [32, 33, 1]. These representations can be found in the decomposition of $\nabla\omega$ and $\nabla\chi$ under SU(4) representations, where ∇ is the Levi-Civita connection. In particular, one schematically has

$$
\begin{aligned}
\nabla_\alpha\omega_{\beta\gamma}+\text{c.c.}&\Longleftrightarrow W_1+W_2\\
\nabla_{\bar\alpha}\omega_{\beta\gamma}+\text{c.c.}&\Longleftrightarrow W_3+W_4\\
\nabla_{\bar\alpha}\chi_{\beta_1\beta_2\beta_3\beta_4}+\text{c.c.}&\Longleftrightarrow W_5 \qquad \text{(C.2)}
\end{aligned}
$$

where W_1, W_2, W_3, W_4 and W_5 have dimensions 4, 20, 20, 4 and 4, respectively. These classes are also contained in $d\omega$ and $d\chi$ as

$$
d\omega^{3,0}+\text{c.c.}\Longleftrightarrow W_1
$$

$$\begin{aligned} d\chi^{3,2} + \text{c.c.} &\Longleftrightarrow W_1 + W_2 \\ d\omega^{2,1} + \text{c.c.} &\Longleftrightarrow W_3 + W_4 \\ d\chi^{4,1} + \text{c.c.} &\Longleftrightarrow W_4 + W_5\,. \end{aligned} \tag{C.3}$$

The class W_1 is chosen such that $W_1 = d\omega^{3,0} + d\omega^{0,3}$. This class can also be represented by a one-form but this is special to the SU(4) structure group. Using this definition for W_1, one can write

$$\nabla_\alpha \omega_{\beta\gamma} = \frac{1}{3}(W_1)_{\alpha\beta\gamma} + (W_2)_{\alpha\beta\gamma}\,. \tag{C.4}$$

This equation can then be considered as the definition of W_2. The class W_4 can be represented by the Lee one-form θ_ω of ω, i.e.

$$W_4 = \theta_\omega = -\star(\star d\omega \wedge \omega)\,. \tag{C.5}$$

We follow the form conventions of [1]. In turn W_3 is defined by the relation

$$d\omega^{2,1} + d\omega^{1,2} = W_3 + \frac{1}{3}\omega \wedge W_4\,. \tag{C.6}$$

Next consider the Lee form of $\mathrm{Re}\chi$

$$\theta_{\mathrm{Re}\chi} = -\frac{1}{4}\star(\star d\mathrm{Re}\chi \wedge \mathrm{Re}\chi)\,. \tag{C.7}$$

The Lee form $\theta_{\mathrm{Re}\chi}$ can be decomposed in terms of the W_4 and W_5 representations. Since we have given a representative of the W_4 representation, we shall set

$$W_5 = \theta_{\mathrm{Re}\chi}\,. \tag{C.8}$$

Let us consider the change of the classes under conformal transformations $ds^2 \to e^{2f} ds^2$ of the metric and changes in the trivialization of the canonical bundle $\mathcal{K}$. The latter are equivalent to transforming $\chi \to e^{i\lambda}\chi$. In particular one finds that

$$\begin{aligned} &W_1^{f,\lambda} = e^{2f} W_1\,, \qquad W_2^{f,\lambda} = e^{2f} W_2\,, \qquad W_3^{f,\lambda} = e^{2f} W_3\,, \\ &W_4^{f,\lambda} = W_4 + 6df\,, \qquad W_5^{f,\lambda} = W_5 - 4df - d_I\lambda\,. \end{aligned} \tag{C.9}$$

The only class that depends on the trivialization of the canonical bundle is W_5.

C.2 G_2-structures in eight dimensions

The G_2 geometry of eight-dimensional manifolds is characterized by a G_2-invariant three-form φ and a G_2 invariant non-vanishing one-form Z, such that

$$g(Z,Z) = 1\,, \qquad {}^*\varphi \wedge Z = 0\,, \qquad \varphi \wedge {}^*\varphi = -7\,d\mathrm{vol}\,. \tag{C.10}$$

In particular ${}^*\varphi = Z \wedge \star\varphi$, where $\star\varphi$ is the standard G_2 invariant four-form. The tangent bundle of the eight-dimensional manifolds decomposes as $TM = \mathbb{R} \oplus E$, where E is a rank

7 vector bundle. One can choose an orthonormal basis such that the metric on M can be written as

$$ds^2 = (e^1)^2 + \delta_{ij} e^i e^j\,, \qquad Z = e_1\,, \qquad i,j = 2,\ldots,8\,, \tag{C.11}$$

To find the different G_2-structures in an eight-dimensional manifold, one can use the method proposed in [34]. The intrinsic torsion of such a manifold decomposes in terms of ten irreducible G_2 representations. So there are 2^{10} G_2-structures in an eight-dimensional manifold. These representations can be found by decomposing ∇Z and $\nabla\varphi$ in terms of G_2 representations. In particular, one has that

$$\begin{aligned} \nabla_1 Z_i &\Longleftrightarrow X \\ \nabla_1 \varphi_{ijk} &\Longleftrightarrow W \\ \nabla_i Z_j &\Longleftrightarrow X_1 + X_2 + X_3 + X_4 \\ \nabla_i \varphi_{jkl} &\Longleftrightarrow W_1 + W_2 + W_3 + W_4 \end{aligned} \tag{C.12}$$

where X and W both have dimension 7, X_1, X_2, X_3 and X_4 have dimensions 1, 7, 14 and 27, respectively, and similarly for W_1, W_2, W_3 and W_4. It is straightforward to find X, W, X_1, X_2, X_3, X_4, W_1, W_2, W_3 and W_4 in terms of ∇Z and $\nabla\varphi$.

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